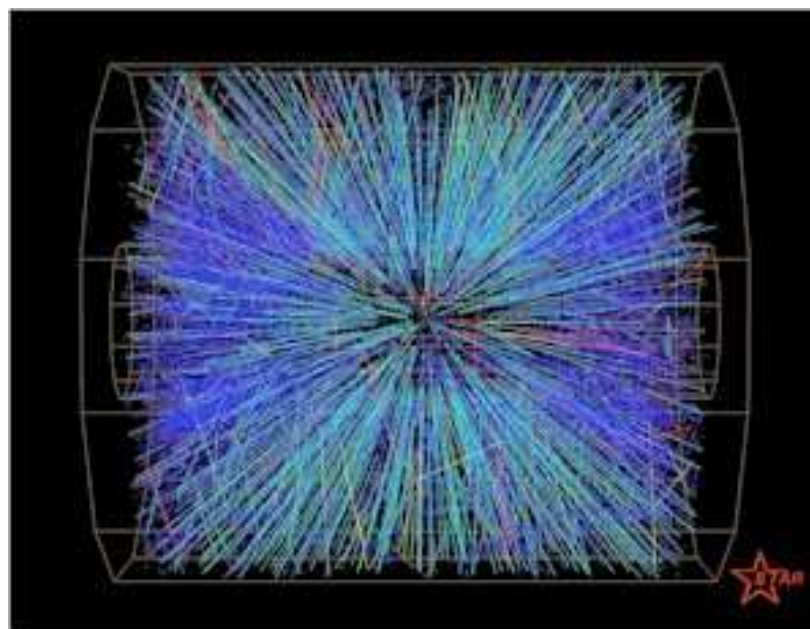
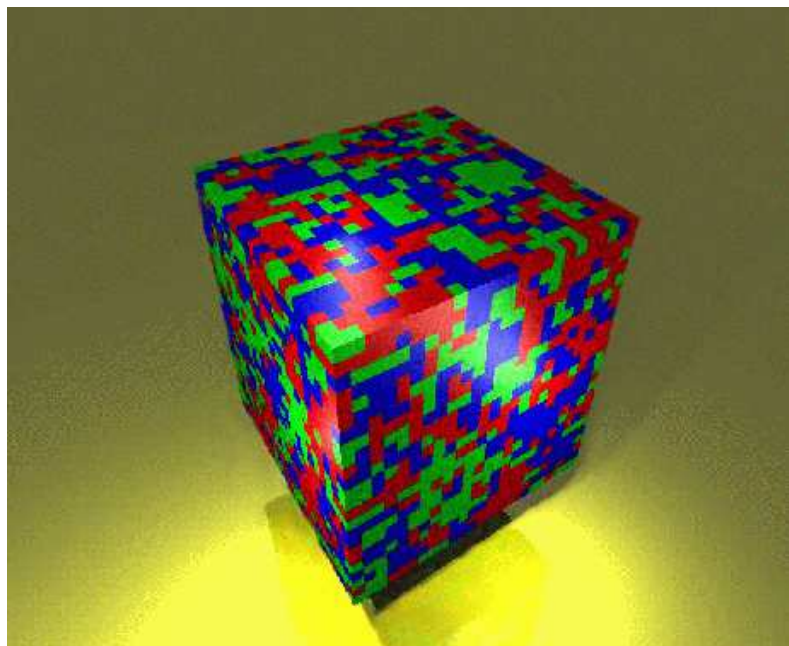
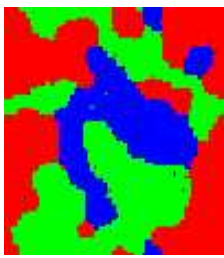
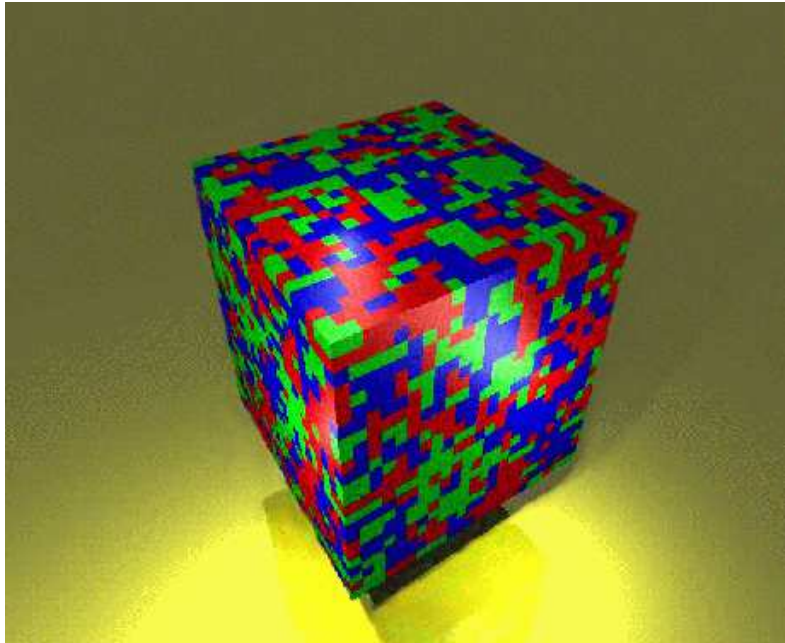


Lattice Gauge Theory and Heavy Ion Collisions



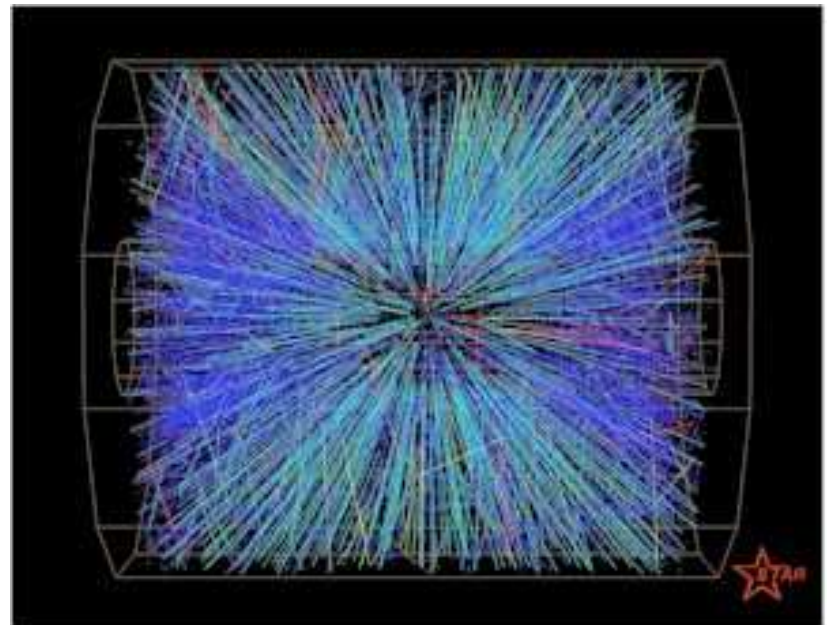


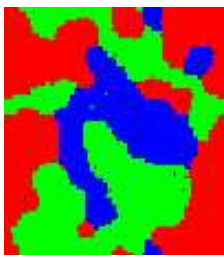
Lattice Gauge Theory and Heavy Ion Collisions



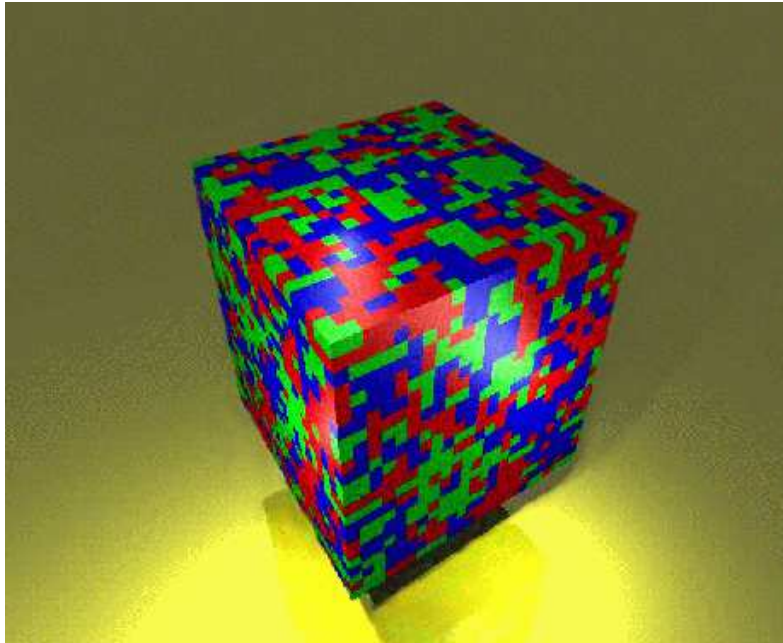
LGT:

- equilibrium thermodynamics of QCD;
- formulated in terms of basic degrees of freedom: quarks and gluons;
- observables expressed in terms of temperature and chemical potential





Lattice Gauge Theory and Heavy Ion Collisions

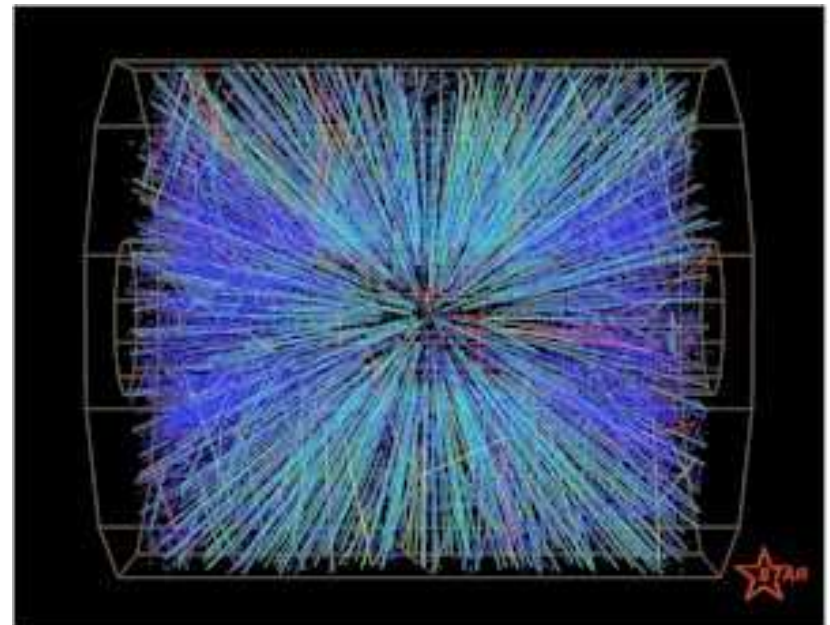


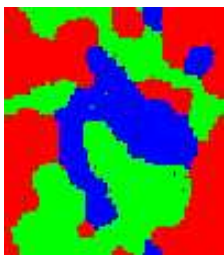
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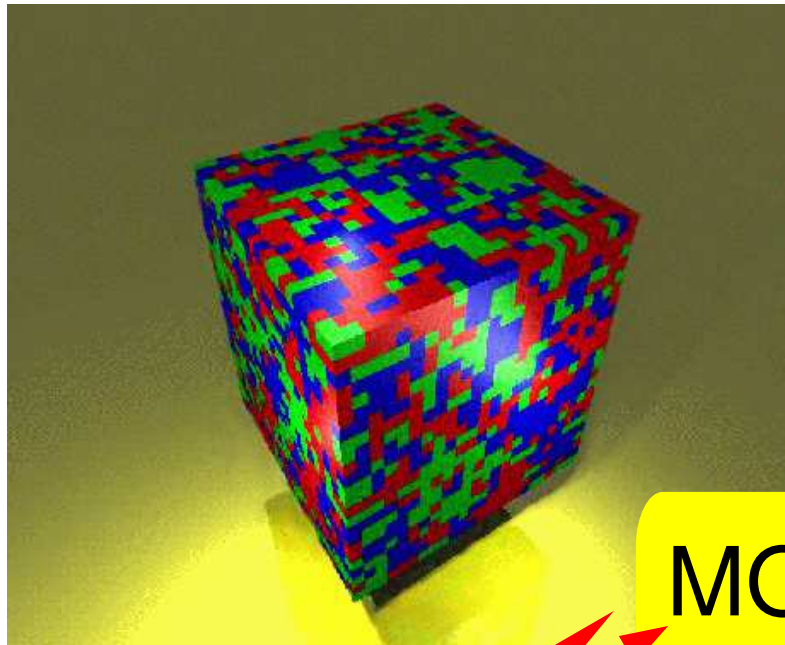
HIC:

- evolution of a dense interacting medium described by QCD;
- observable properties in terms of hadrons, leptons and photons;
- observables parametrized in terms of energy and particle multiplicities





Lattice Gauge Theory and Heavy Ion Collisions



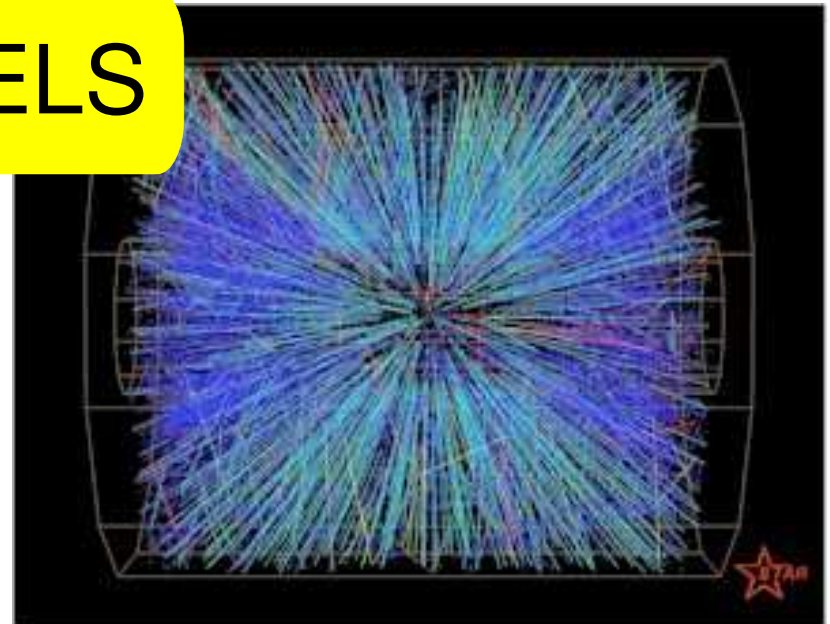
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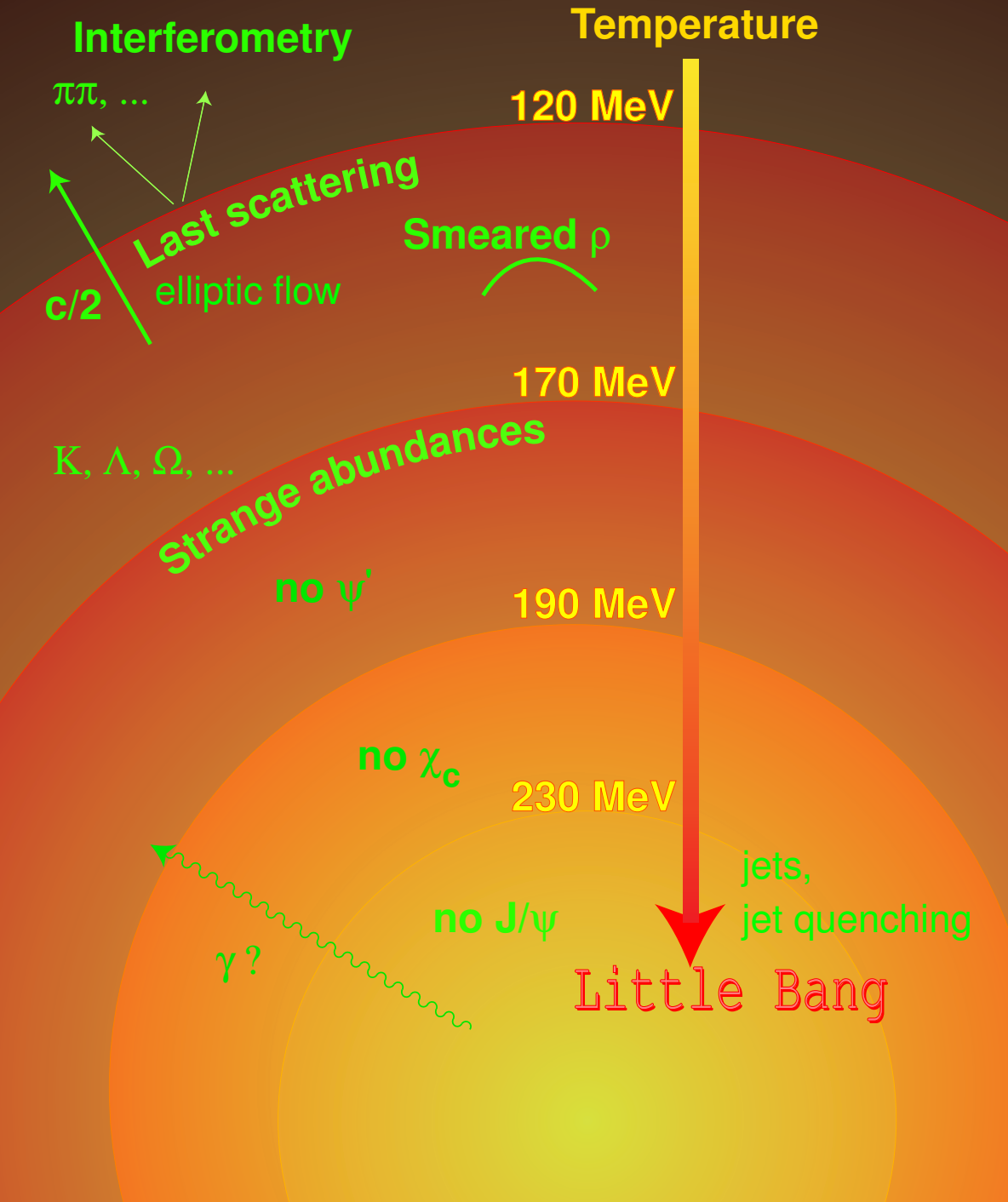
MODELS

LGT:

- equilibrium thermodynamics of QCD;
- formulated in terms of basic degrees of freedom: quarks and gluons;
- observables expressed in terms of temperature and chemical potential



Towards A New State of Matter



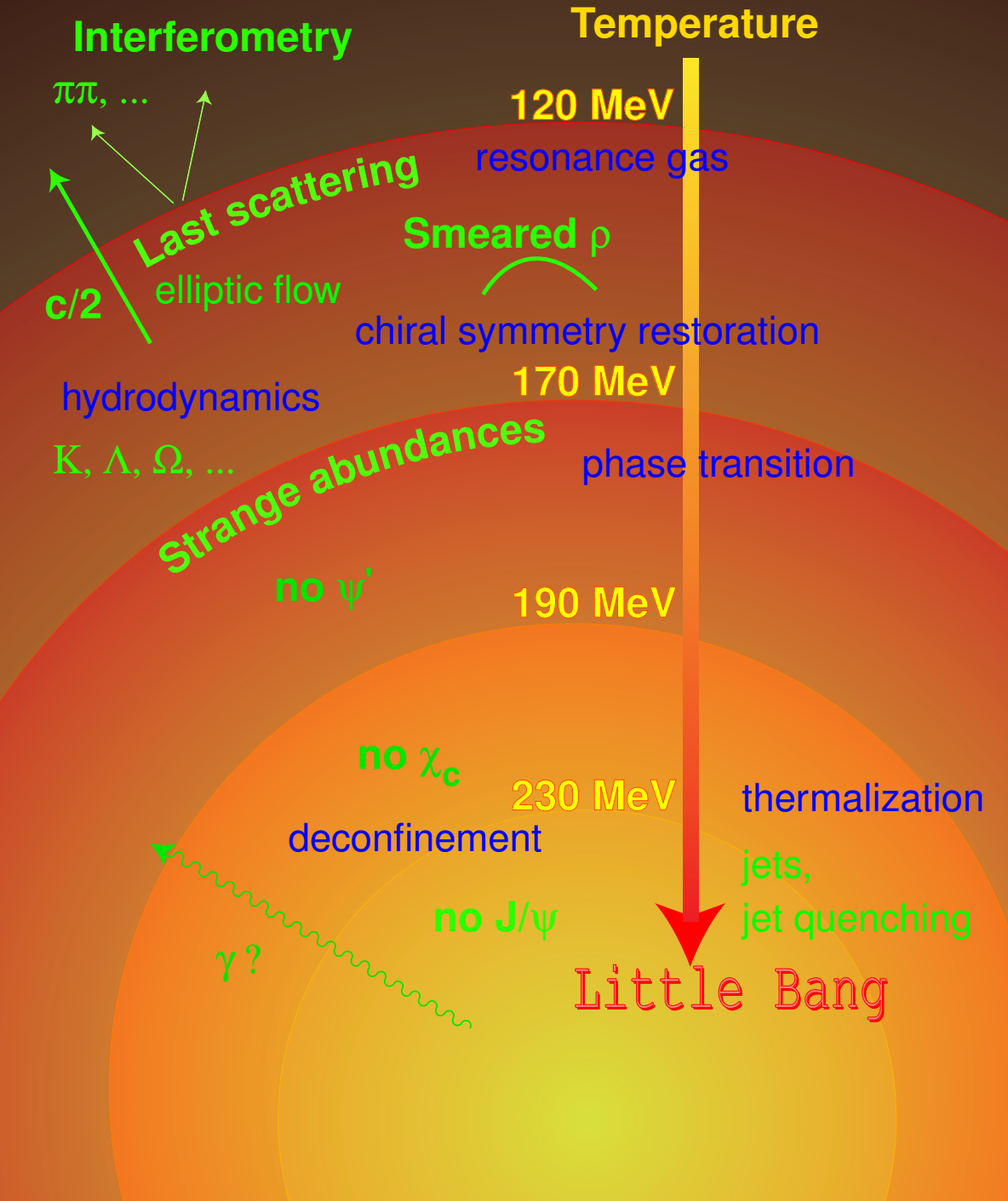
Where lattice calculations do/will contribute to the

development of theoretical concepts

and the

analysis of experimental observables

Towards A New State of Matter



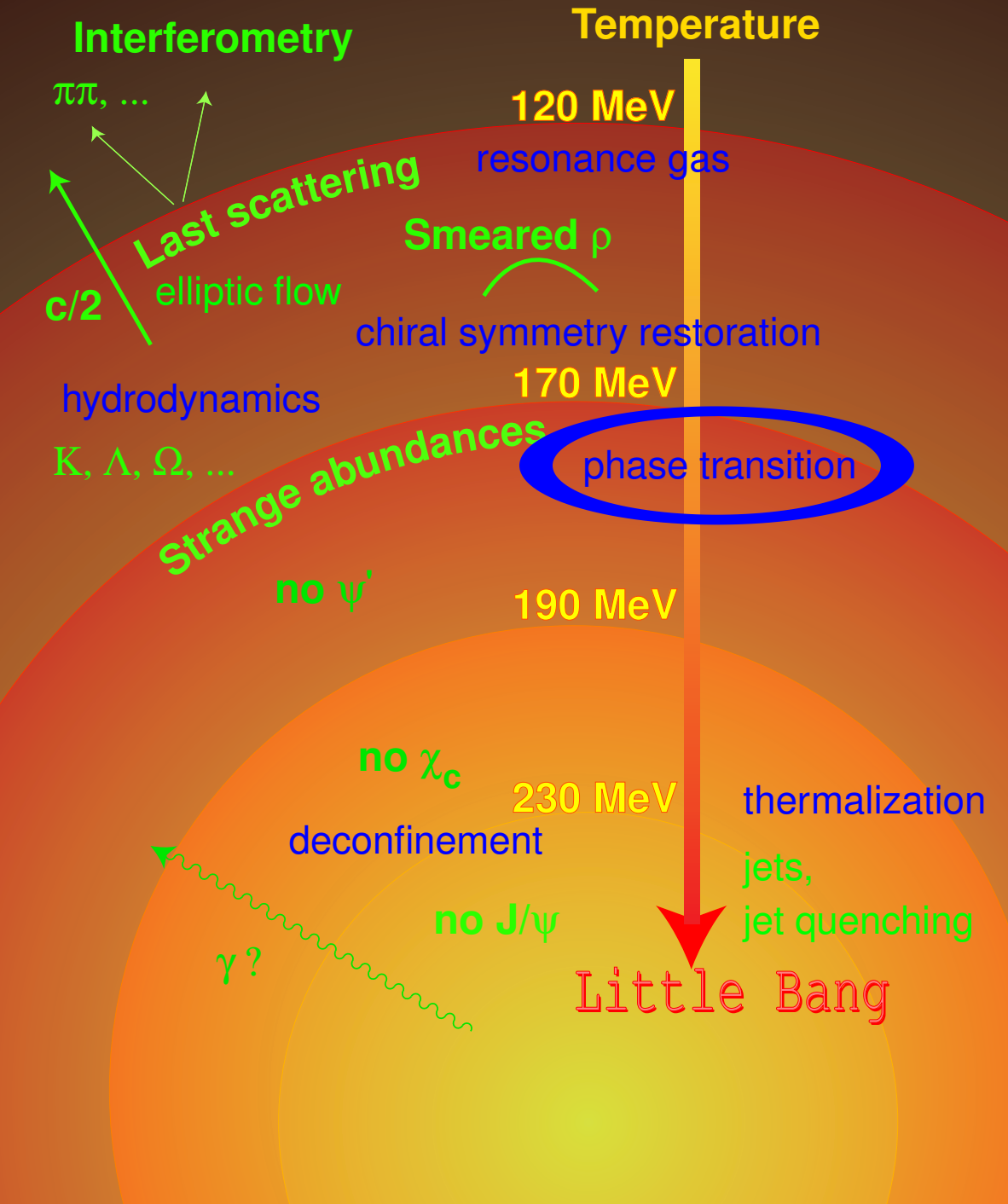
Where lattice calculations do/will contribute to the

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Towards A New State of Matter



Where lattice calculations do/will contribute to the

development of theoretical concepts

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analysis of experimental observables

$$T_c, \epsilon_c$$

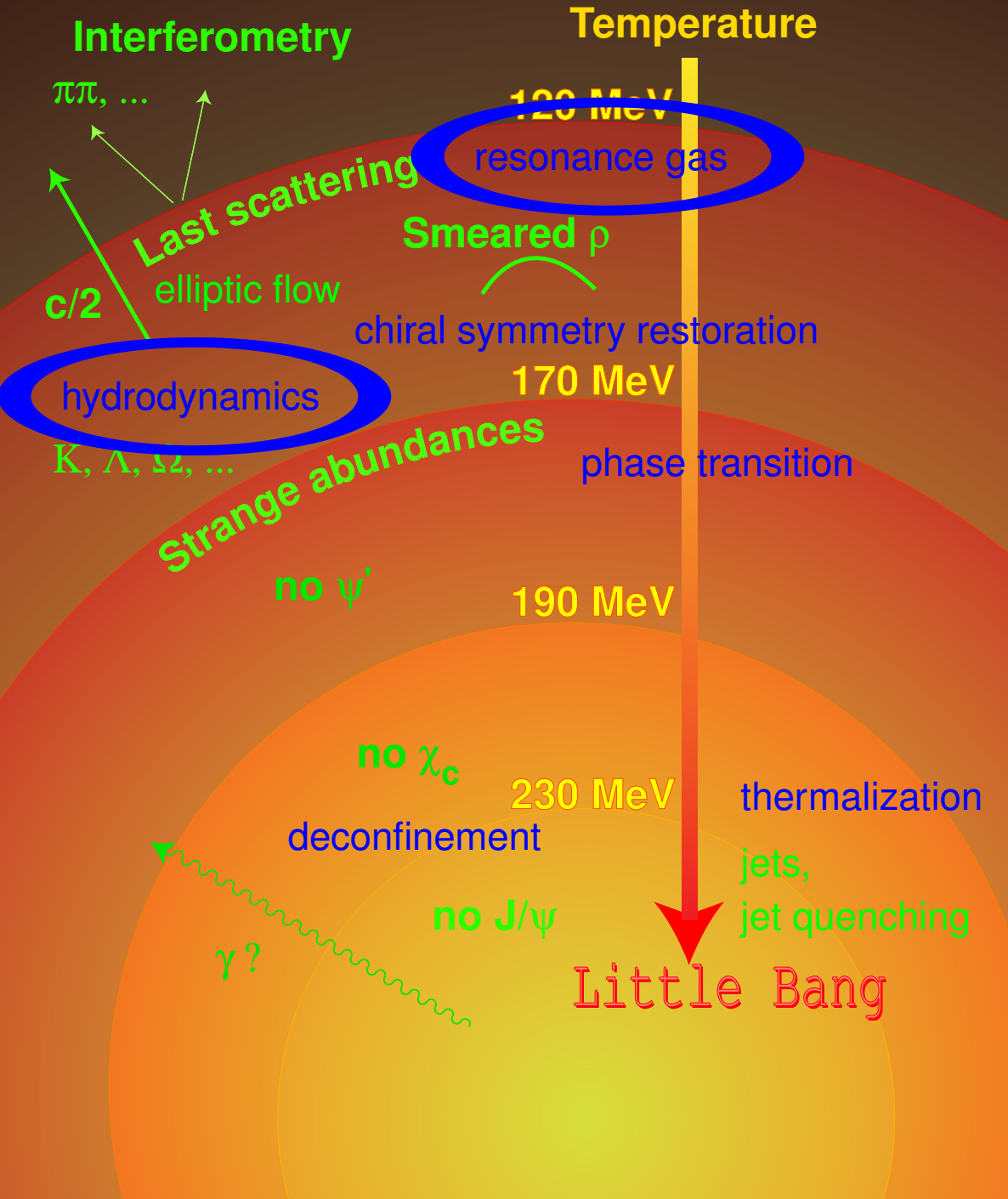
phase diagram in the (T, μ_B) -plane;

$\mu \simeq 0$: RHIC (LHC)

$\mu > 0$: SPS (GSI future)

chiral critical point

Towards A New State of Matter



Where lattice calculations do/will contribute to the

development of theoretical concepts

and the

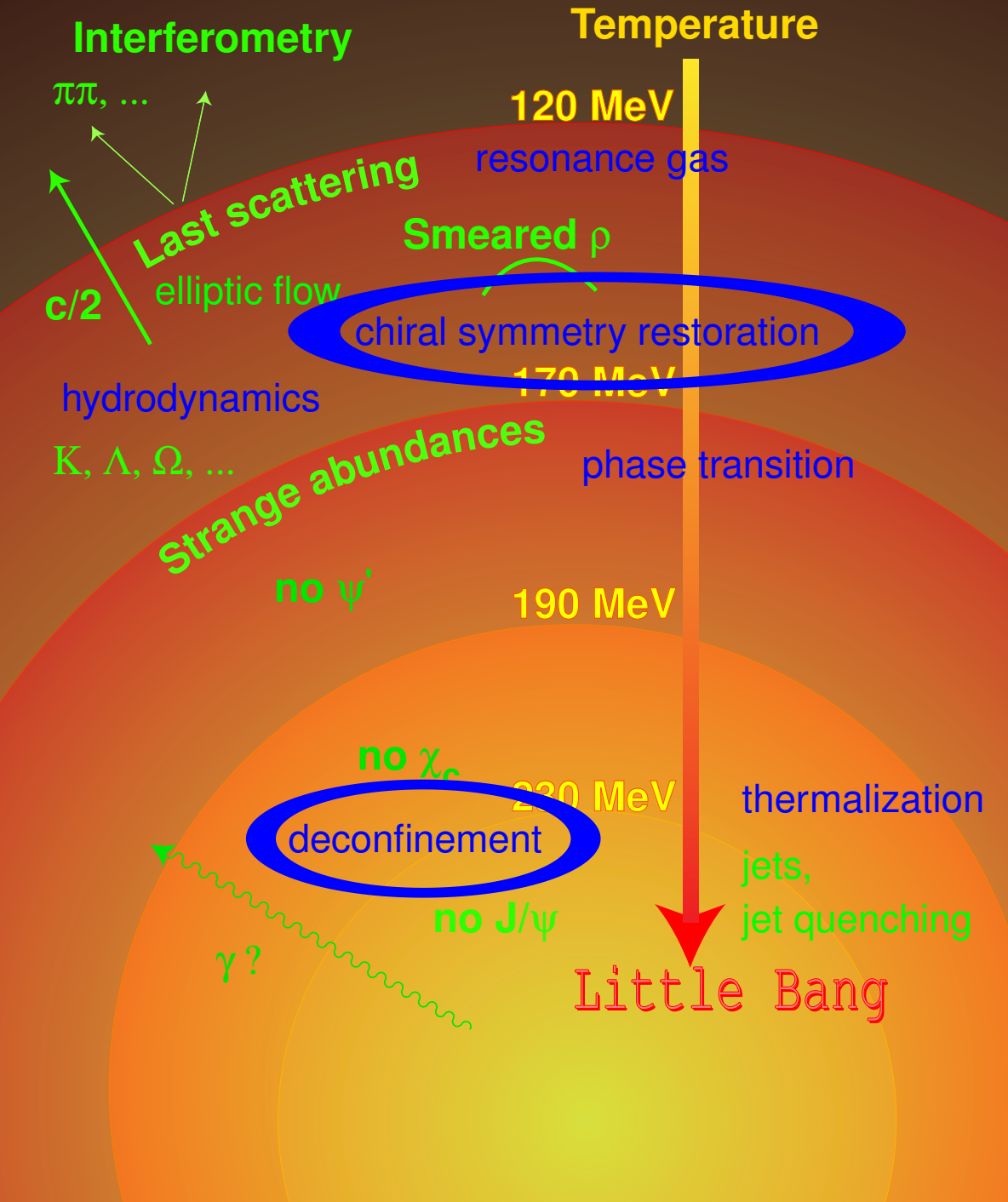
analysis of experimental observables

EoS

energy density, pressure, velocity of sound,...; susceptibilities (baryon number fluctuations);

strangeness contribution

Towards A New State of Matter



Where lattice calculations do/will contribute to the

development of theoretical concepts

and the

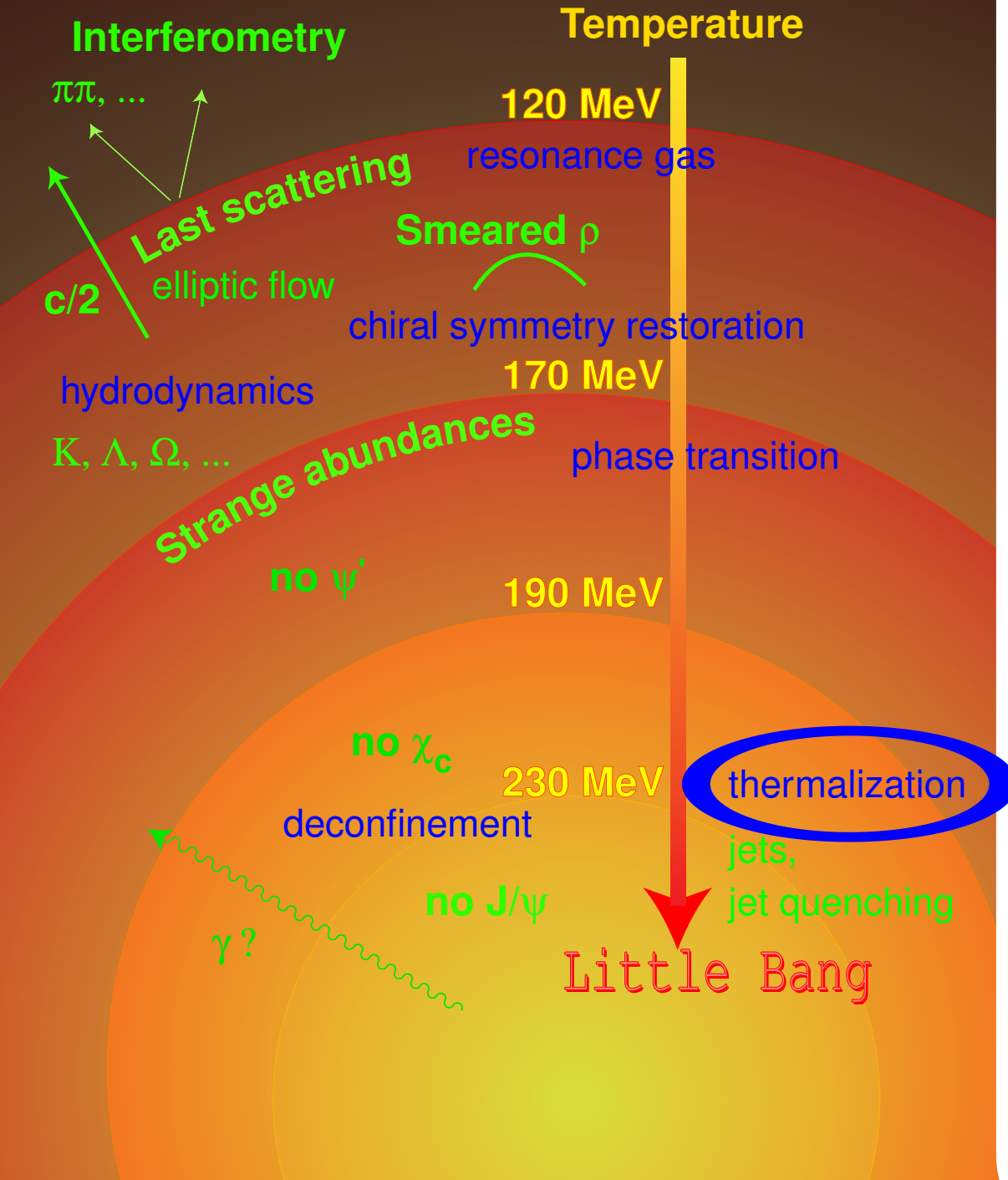
analysis of experimental observables

In – medium hadron properties

heavy quark potential, screening;
charmonium spectroscopy;
light quark bound states;

thermal dilepton rates

Towards A New State of Matter



Where lattice calculations do/will contribute to the

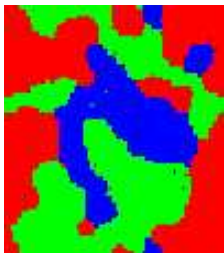
development of theoretical concepts

and the

analysis of experimental observables

short vs. long distance physics

running coupling constant;
transport coefficients



Progress in lattice calculations... depends on...

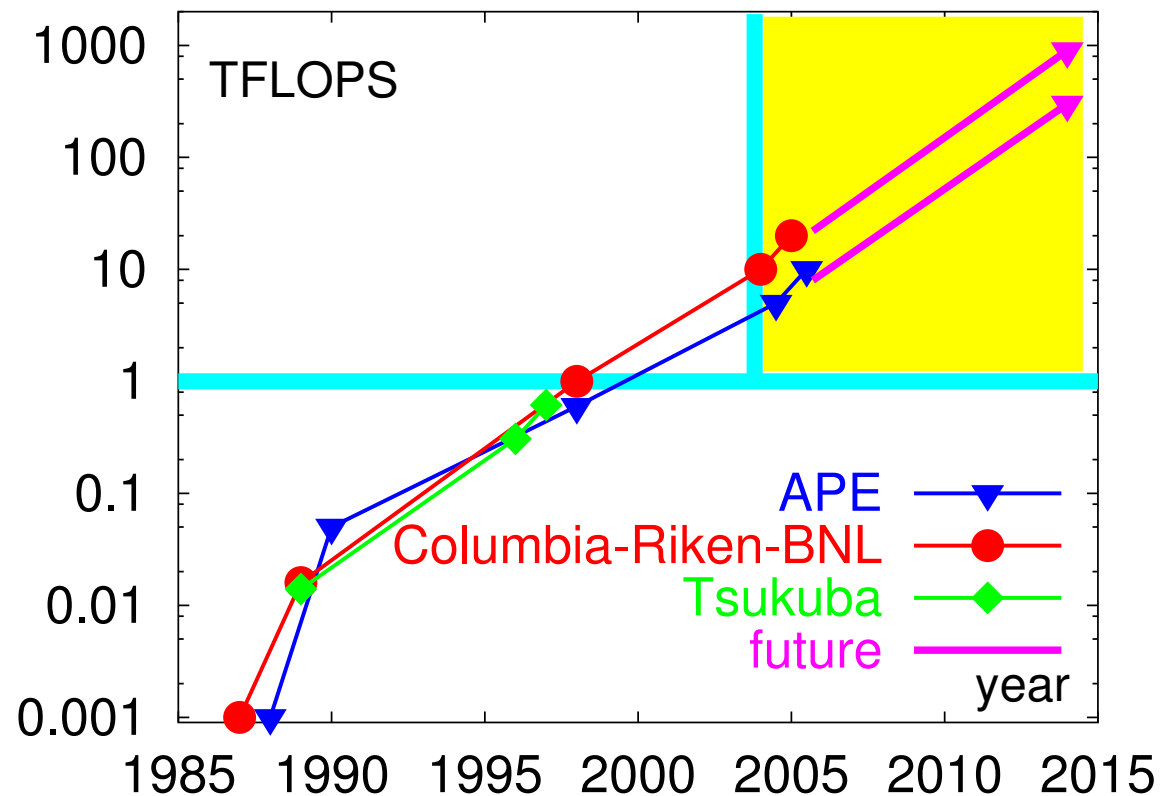
● development of (special purpose) computer hardware

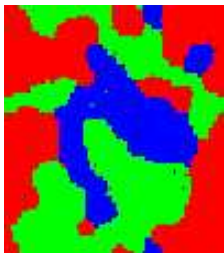


development of
special purpose
computer hardware



towards PETAFLUPS
computing





Progress in lattice calculations... depends on...

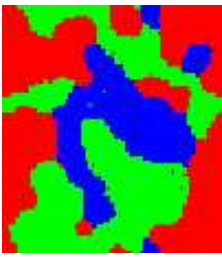
- development of (special purpose) computer hardware
- progress in algorithm development
-

1987 invention of Hybrid Monte Carlo Algorithm

early '90s development of various preconditioning schemes

late '90s new algorithms: polynomial / shifted HMC; multi-boson algorithm

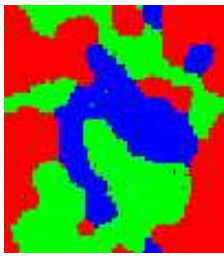
1987 - 2004: gain of factor 15 - 20 from algorithm development



Progress in lattice calculations... depends on...

- development of (special purpose) computer hardware
- progress in algorithm development
- new ideas, new conceptual developments!!!

- 1988/89 multi-parameter Ferrenberg-Swendsen reweighting
⇒ accurate location of transition points, scaling analysis
- 1996 Non-perturbative definition of bulk thermodynamics
⇒ integral method for reliable pressure calculations
- 1999 Maximum Entropy Method (MEM) for QCD
⇒ spectral functions, in-medium properties of hadrons
- 2002 reweighting and Taylor expansion techniques for $\mu > 0$
⇒ QCD phase diagram at finite baryon density



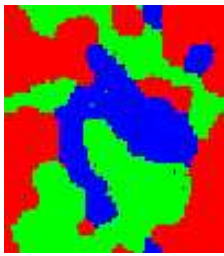
Outlook: Next generation computers for lattice gauge theory



today:

APEmille

so far the only dedicated
large-scale computer installation used
predominantly for QCD thermodynamics
exists in Bielefeld: 120 GFlops



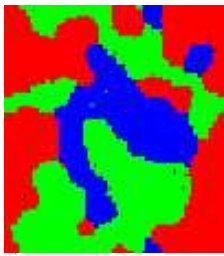
Outlook: Next generation computers for lattice gauge theory

QCDOC and apeNEXT

2004:

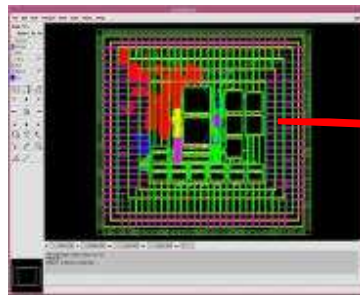
QCD thermodynamics on the next generation of special purpose
dedicated QCD computers

installations with (10-20) TFlops peak speed are planned
in the USA and Europe

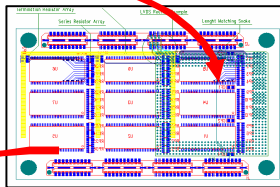


apeNEXT: Next generation of APE computers

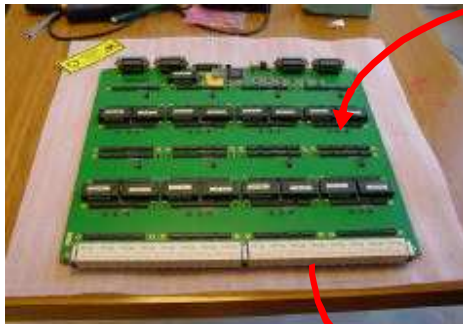
Assembling apeNEXT...



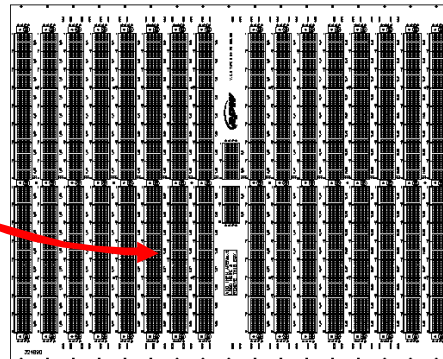
J&T Asic



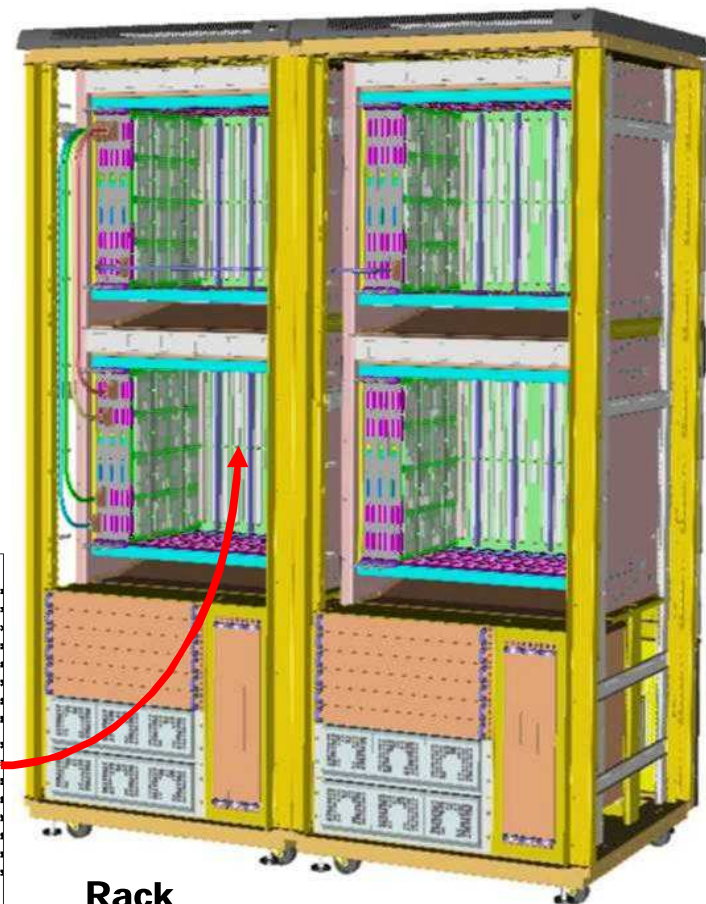
J&T module



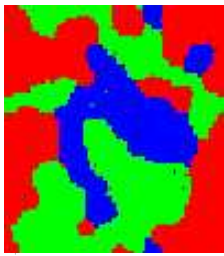
PB



BackPlane

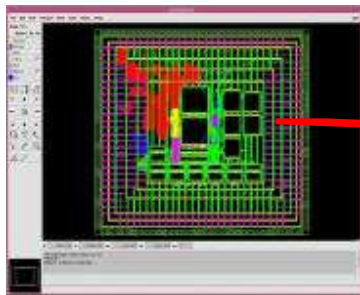


Rack

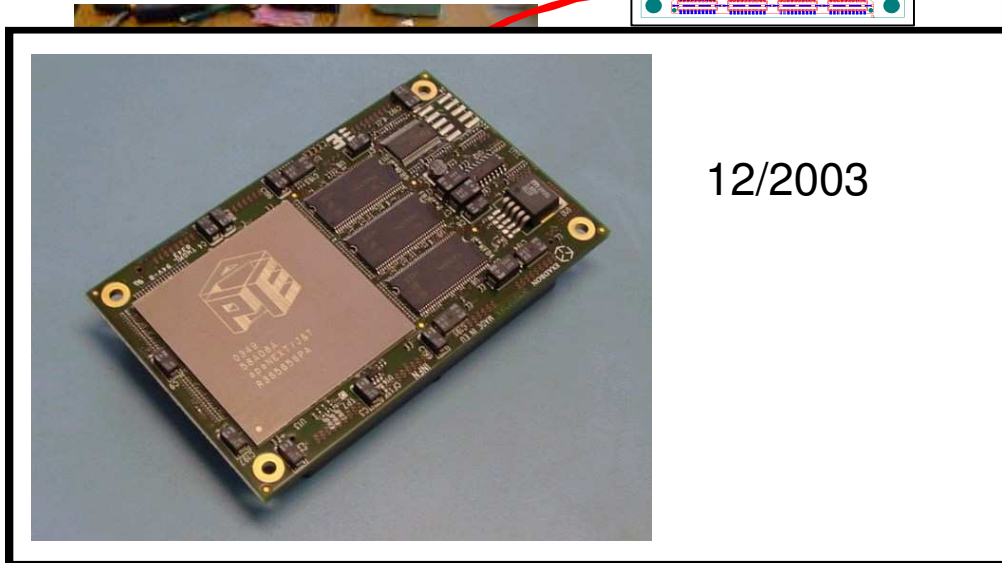
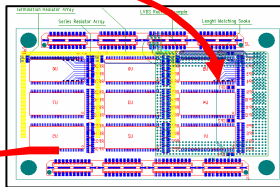


apeNEXT: Next generation of APE computers

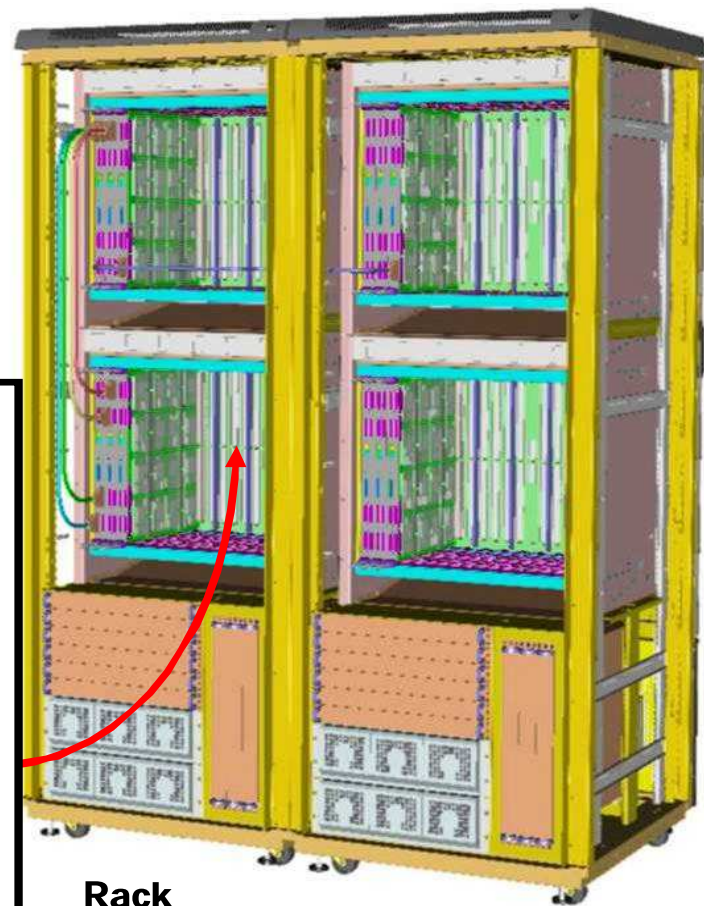
Assembling apeNEXT...



J&T Asic

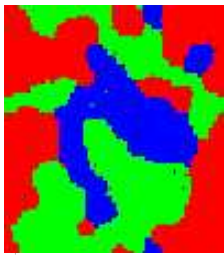


12/2003



Rack

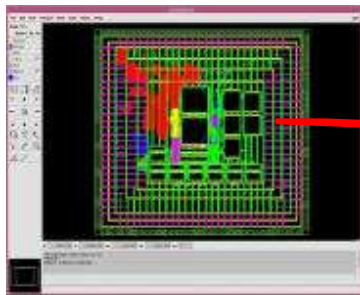
BackPlane



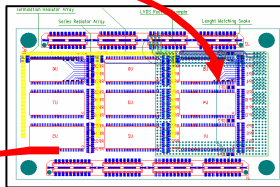
apeNEXT: Next generation of APE computers



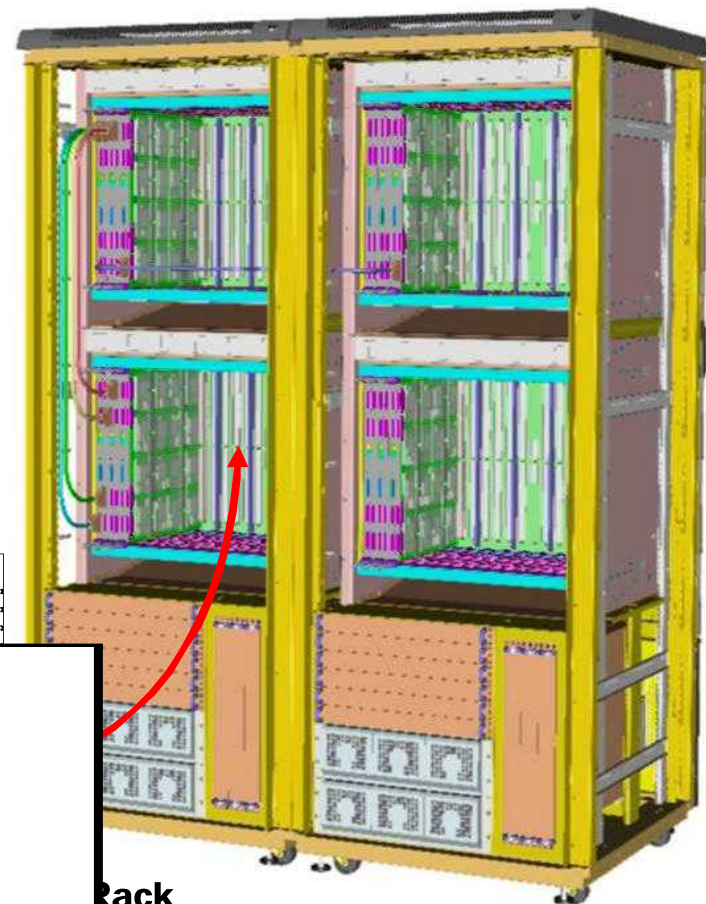
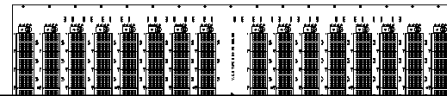
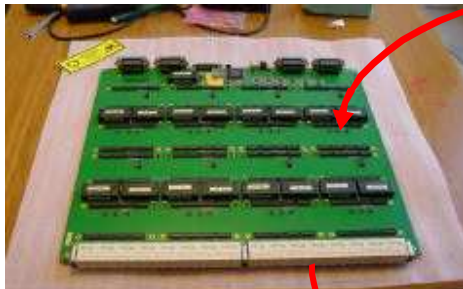
Assembling apeNEXT...



J&T Asic



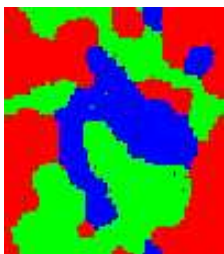
J&T module



Rack

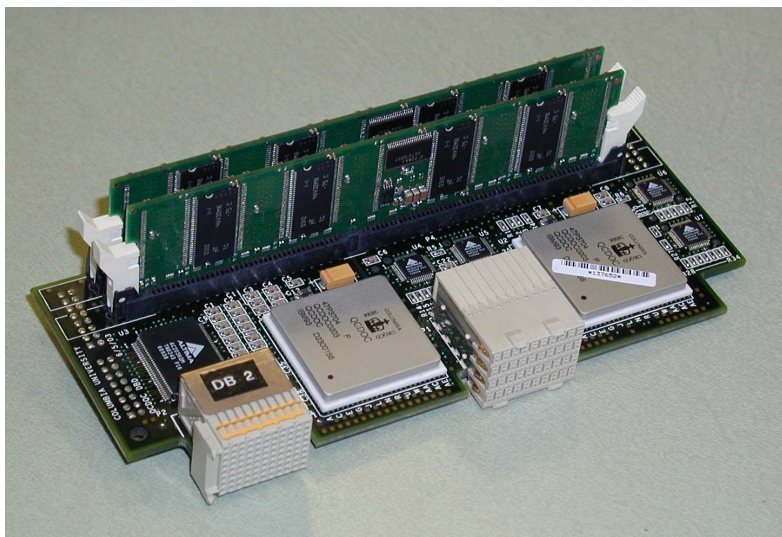
- first chips Dec. 2003
- two 0.8 TFlops prototypes ~ summer 2004
- first 3 TFlops installations in 2005

BackPlane



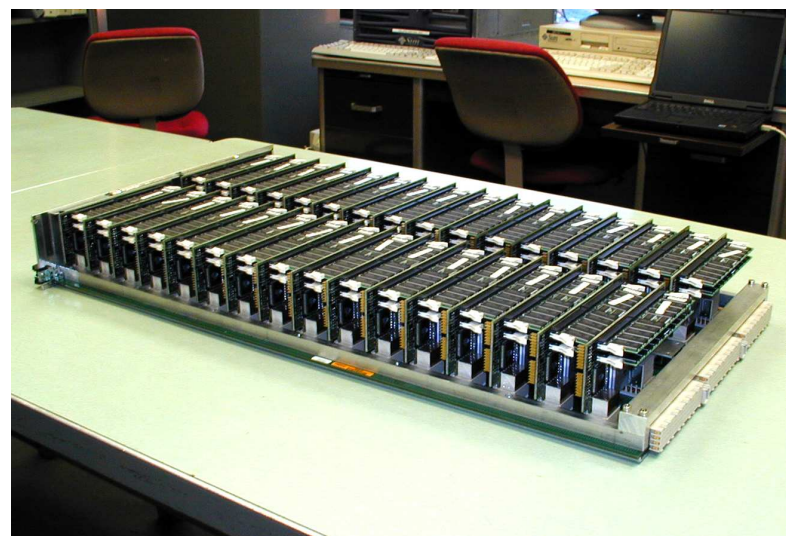
QCDOC: Next generation of Columbia-RIKEN computer

Columbia – RIKEN – UKQCD Collaboration

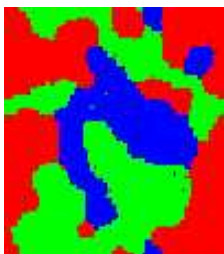


2 – node daughter card

- prototypes exist since 07/2003



64 – node mother board



QCDOC: Next generation of Columbia-RIKEN computer

Columbia – RIKEN – UKQCD Collaboration

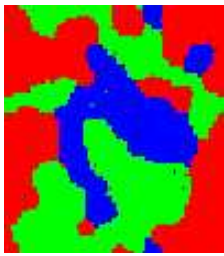


512 – node machine : (360 – 450) GFlops

- currently being debugged prototype (05/2004):
0.25 Tbyte memory; 6 Gbit/sec Ethernet I/O bandwidth

QCDOC computing center at BNL :

- 10 TFlops machine for RBRC: ~ autumn 2004
- 10 TFlops machine for american LGT community: ~ early 2005
- ... larger installations possible and needed!



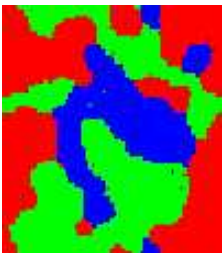
Bulk Thermodynamics: What do we (want to) know?

$\mu = 0$:

- properties of transition in 2 , $(2 + 1)$ -flavor QCD:
crossover or phase transition, deconfinement vs. chiral symmetry restoration, universality, ...
- T_c, ϵ_c, EOS :
confront resonance gas, quasi-particle gas, high-T pert. theory, HTL-resummation, ... with lattice calculations

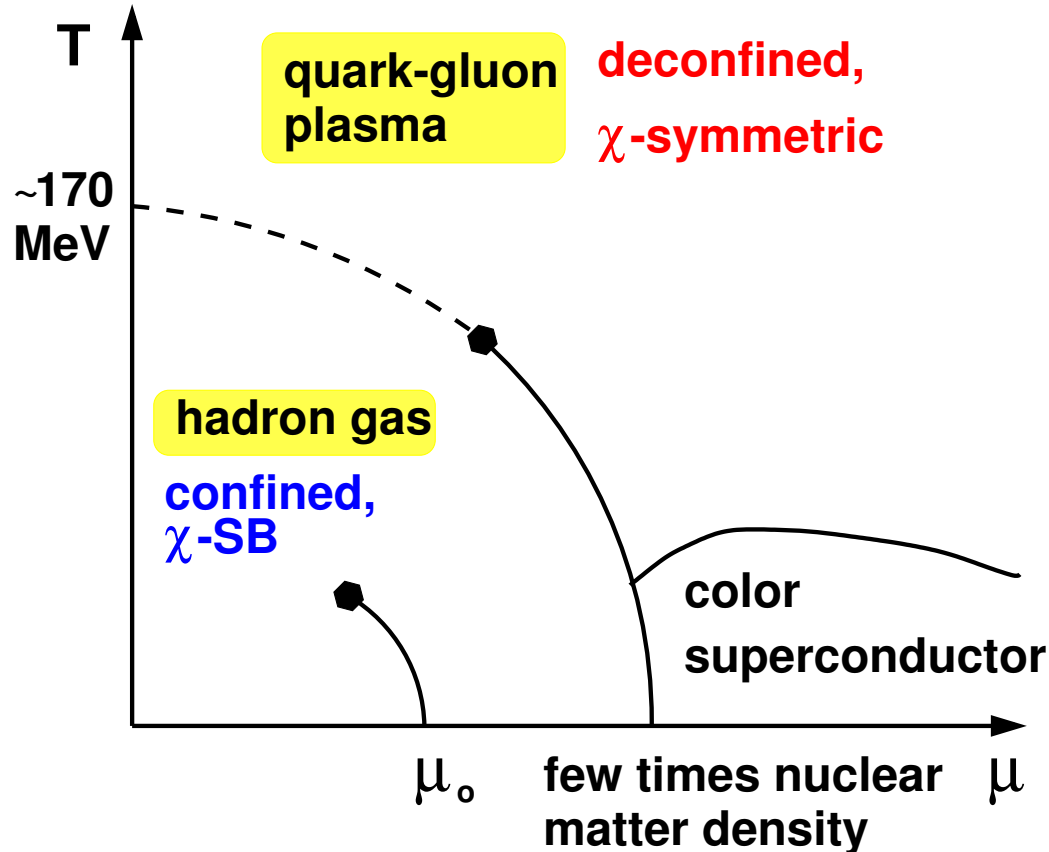
$\mu > 0$:

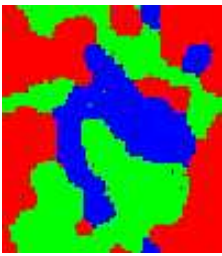
- $T_c(\mu) \Leftrightarrow T_{\text{freeze}}(\mu)$:
location of the chiral critical point, direct evidence for 1^{st} order regime;
density fluctuations; $T_c(\mu) \equiv T_{\text{freeze}}$?



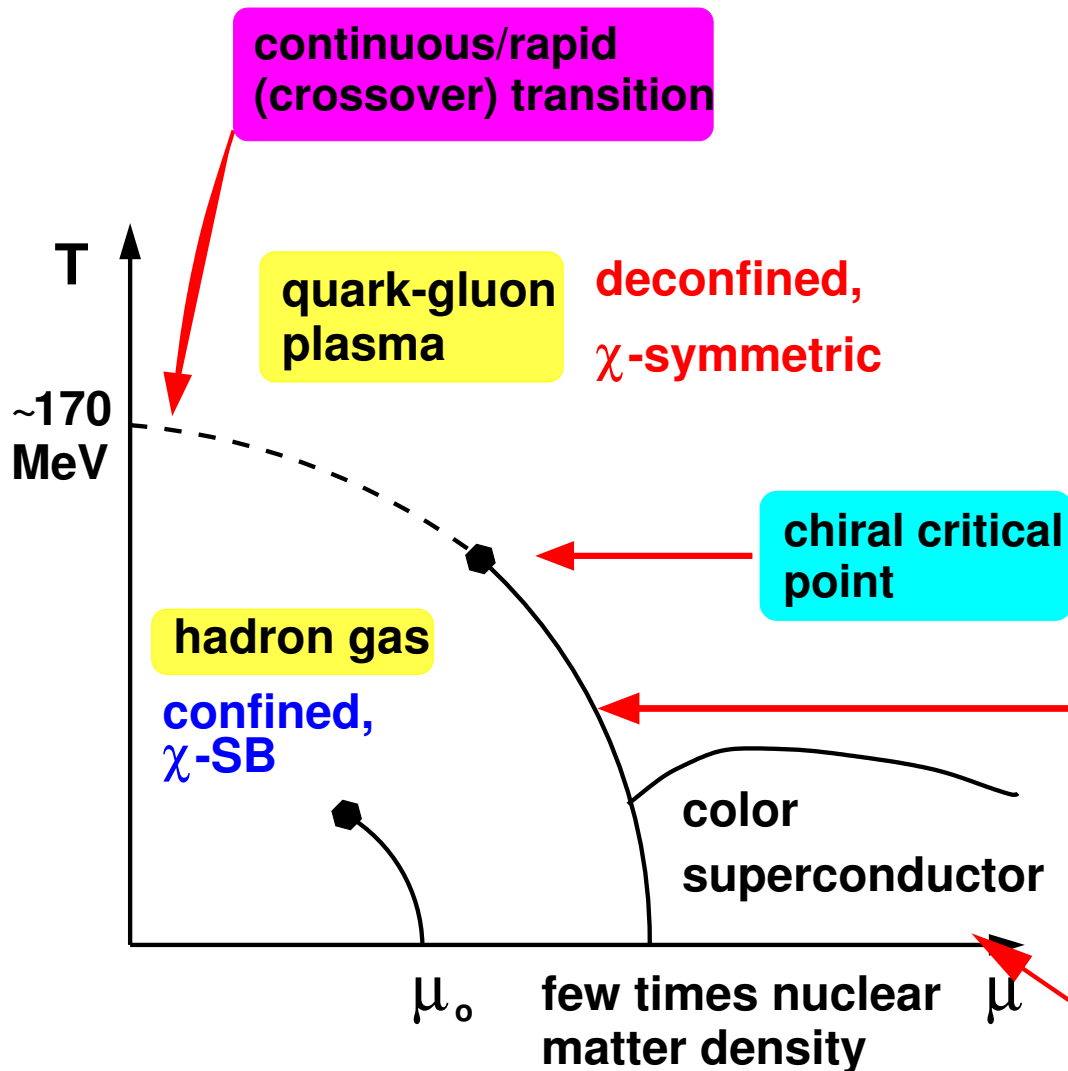
Critical behavior in hot and dense matter: phase diagram

crossover vs.
phase transition





Critical behavior in hot and dense matter: phase diagram



continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170 \text{ MeV}$$

$$\epsilon_c \simeq 0.7 \text{ GeV}/\text{fm}^3$$

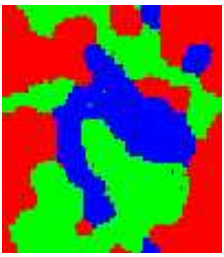
2nd order phase transition; Ising universality class

$$T_c(\mu) \text{ under investigation}$$

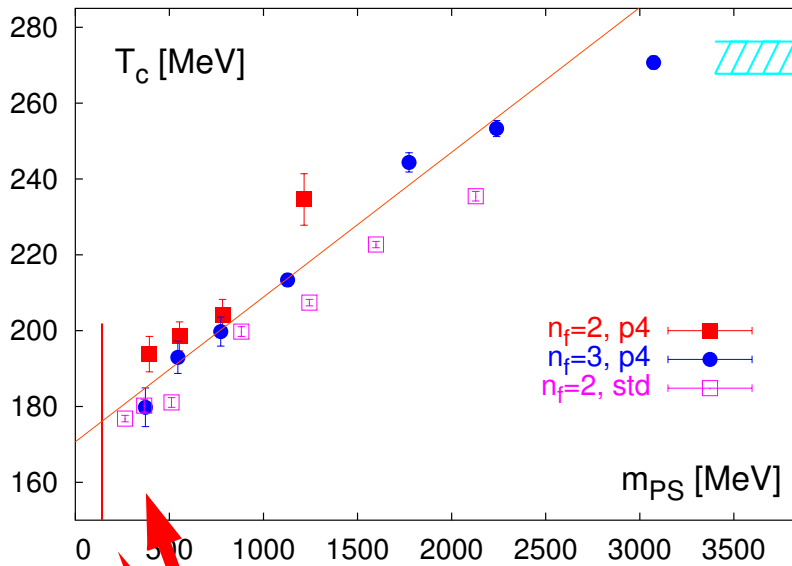
1st order phase transition ???

expected - however, so far no direct evidence from lattice QCD

attractive 1-gluon exchange \Rightarrow qq-condensates



Critical temperature, equation of state



$$T_c = (173 \pm 8 \pm \text{sys}) \text{ MeV}$$

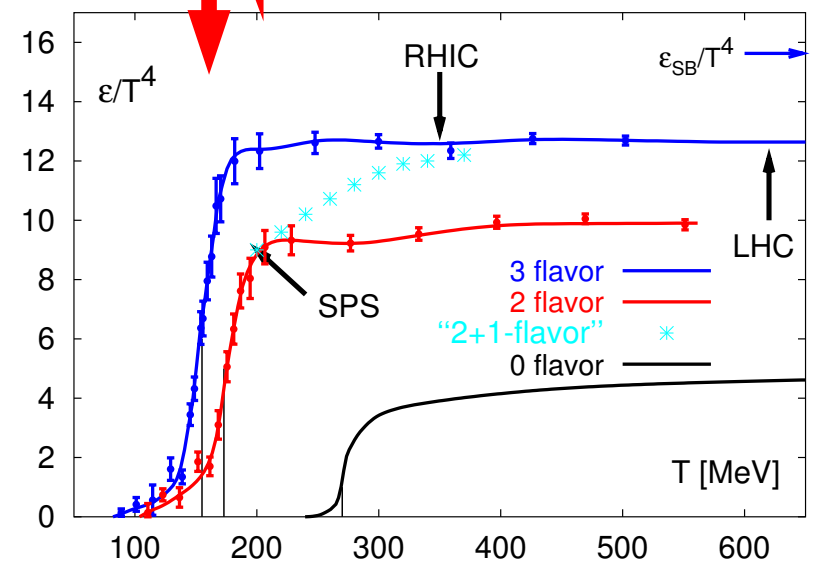
FK, E. Laermann, A. Peikert,
Nucl. Phys. B605 (2001) 579

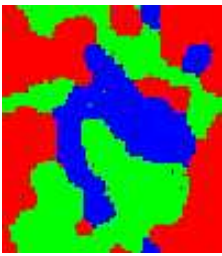
$$T_c = 167(13) [177(11)] \text{ MeV}$$

MILC, hep-lat/0405029

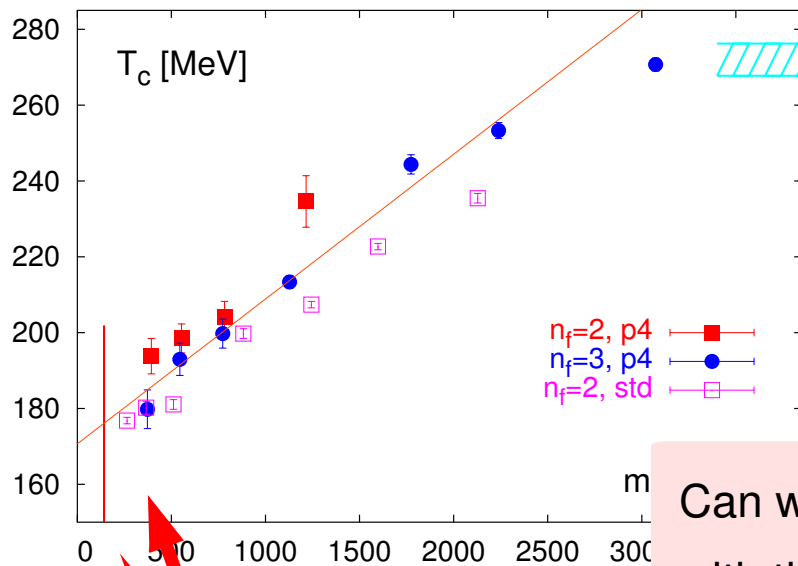
$$\begin{aligned} \epsilon_c &\simeq (6 \pm 2) T_c^4 \\ &\simeq (0.3 - 1.3) \text{ GeV}/\text{fm}^3 \end{aligned}$$

energy density for 0, 2 and 3-flavor QCD





Critical temperature, equation of state



$$\epsilon_c \simeq (6 \pm 2) T_c^4$$

$$\simeq (0.3 - 1.3) \text{ GeV}/\text{fm}^3$$

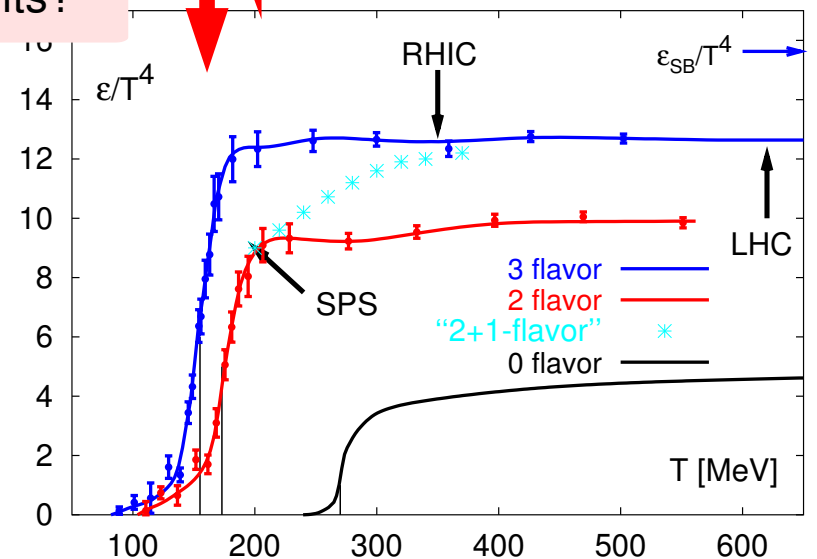
Can we be satisfied with these results?

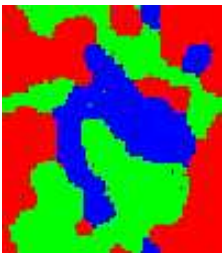
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FK, E. Laermann, A. Peikert,
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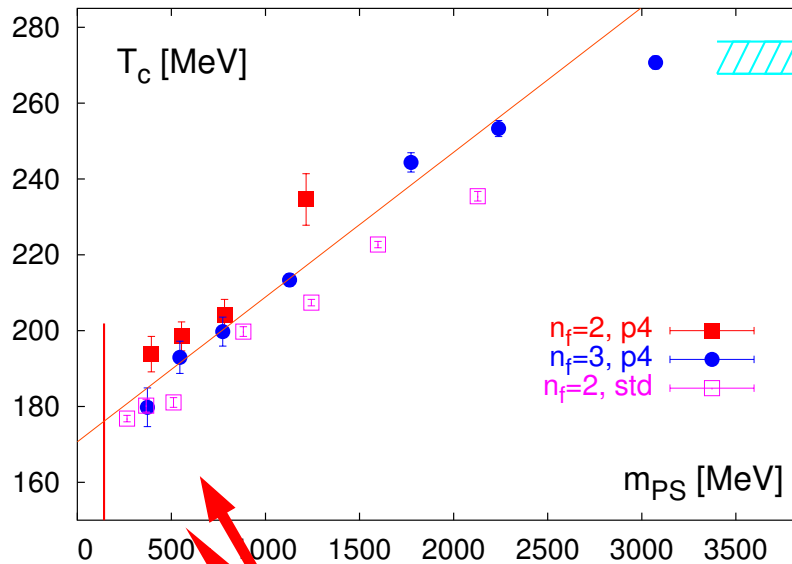
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MILC, hep-lat/0405029





Critical temperature, equation of state



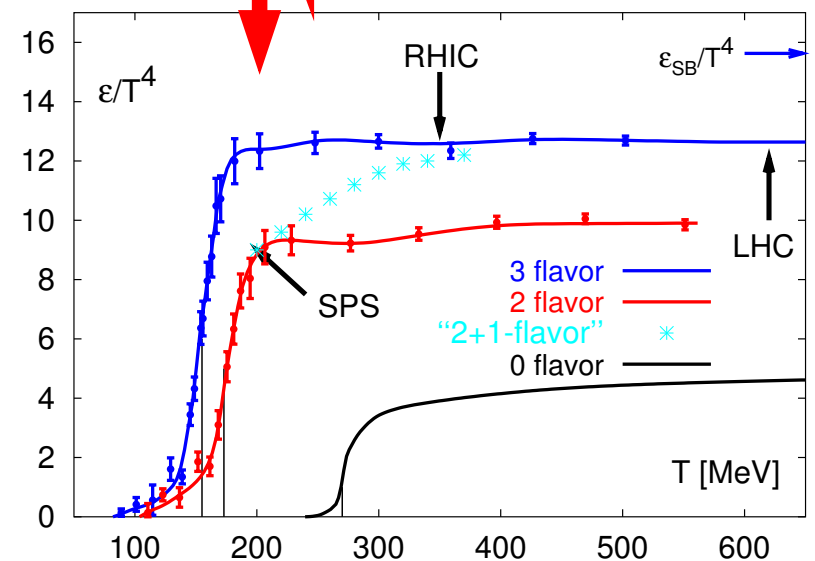
T_c

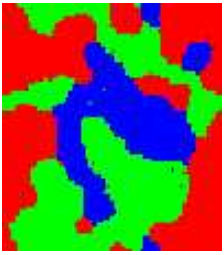
- $m_{PS} \gtrsim 300 \text{ MeV}$ (chiral limit??)
- $a \simeq 0.2 \text{ fm}$ (continuum limit??)
- improved staggered fermions,
 \Rightarrow flavor symmetry breaking
 (need even better fermion actions)

ϵ_c

- $m_{PS} \simeq 770 \text{ MeV}$ (!!!)
- $V \simeq (4 \text{ fm})^3$ (thermodynamic limit)

energy density for 0, 2 and 3-flavor QCD





Extending the phase diagram to non-vanishing chemical potential

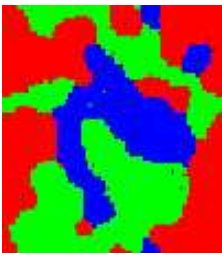
non-zero baryon number density: $\mu > 0$

$$\begin{aligned} Z(V, T, \mu) &= \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E(V, T, \mu)} \\ &= \int \mathcal{D}A \mathcal{D} \det M(\mu) e^{-S_E(V, T)} \end{aligned}$$

↑↑ complex fermion determinant;

long standing problem

⇒ three (partial) solutions for large T , small μ



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

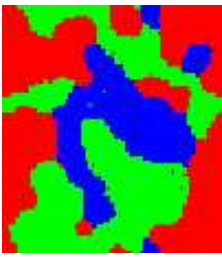
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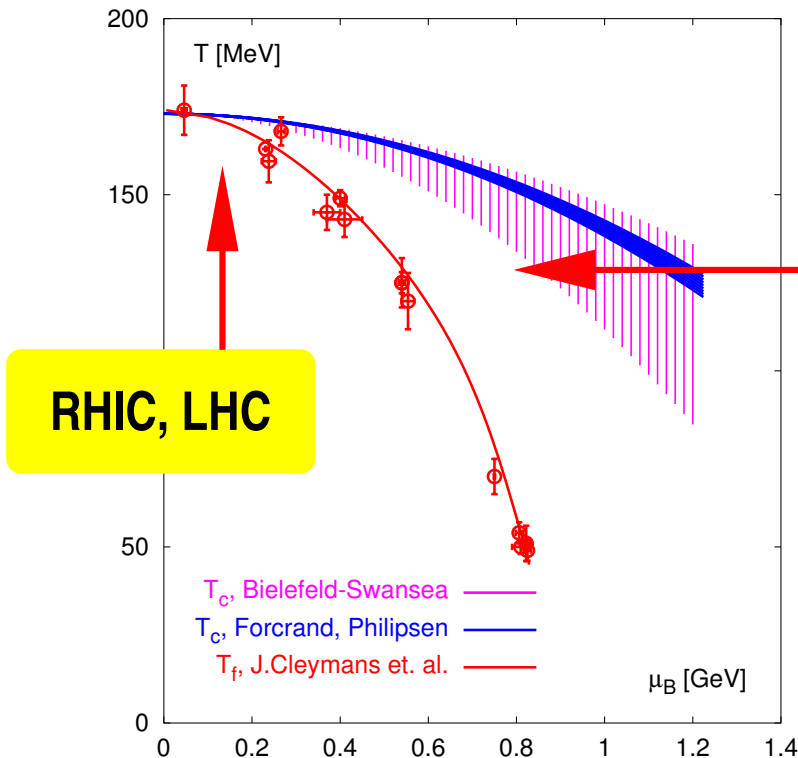
- exact evaluation of $\det M$: works well on small lattices; requires reweighting
Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ ; requires reweighting
C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
- imaginary chemical potential: works well for small μ ; requires analytic continuation
Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$\begin{aligned}
 Z(V, T, \mu) &= \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E(V, T, \mu)} \\
 &= \int \mathcal{D}A \det M(\mu) e^{-S_E(V, T)}
 \end{aligned}$$

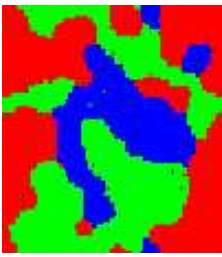


$$\frac{T_c(\mu)}{T_c(0)} : 1 - 0.0056(4)(\mu_B/T)^2$$

deForcrand, Philipsen (imag. μ)

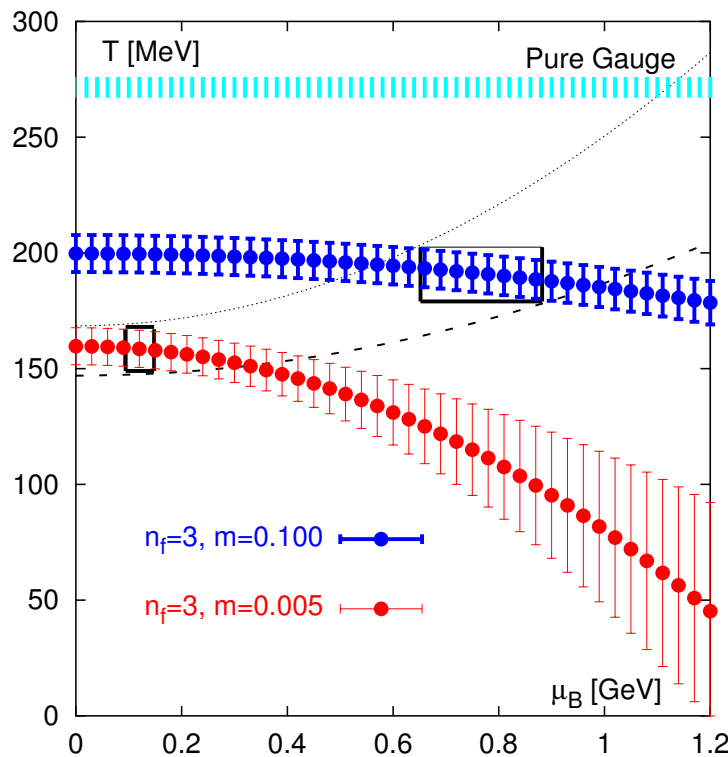
$$1 - 0.0078(38)(\mu_B/T)^2$$

Bielefeld-Swansea
 $(\mathcal{O}(\mu^2)$ reweighting)



Extending the phase diagram to non-vanishing chemical potential

first (exploratory) results on the quark mass dependence of the transition line:



m_q -dependence

(3-flavor QCD)

$$\frac{T_c(\mu)}{T_c(0)} : 1 - 0.025(6)(\mu_q/T)^2, \quad ma = 0.1$$

$$1 - 0.114(46)(\mu_q/T)^2, \quad ma = 0.005$$

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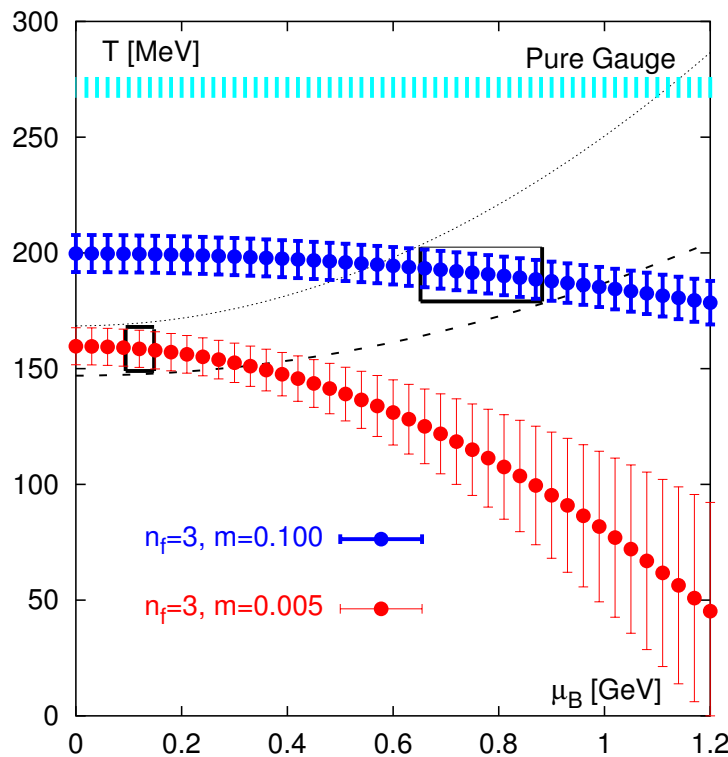
(hep-lat/0309116, Lattice 2003)

a systematic analysis of
cut-off effects, scaling violations
AND volume + truncation effects
still needs to be done



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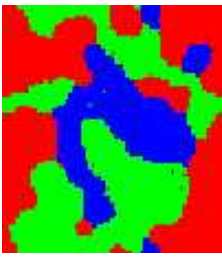
Bielefeld-Swansea

(hep-lat/0309116, Lattice 2003)

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$$T_c(\mu) \equiv T_{\text{freeze}} ?$$

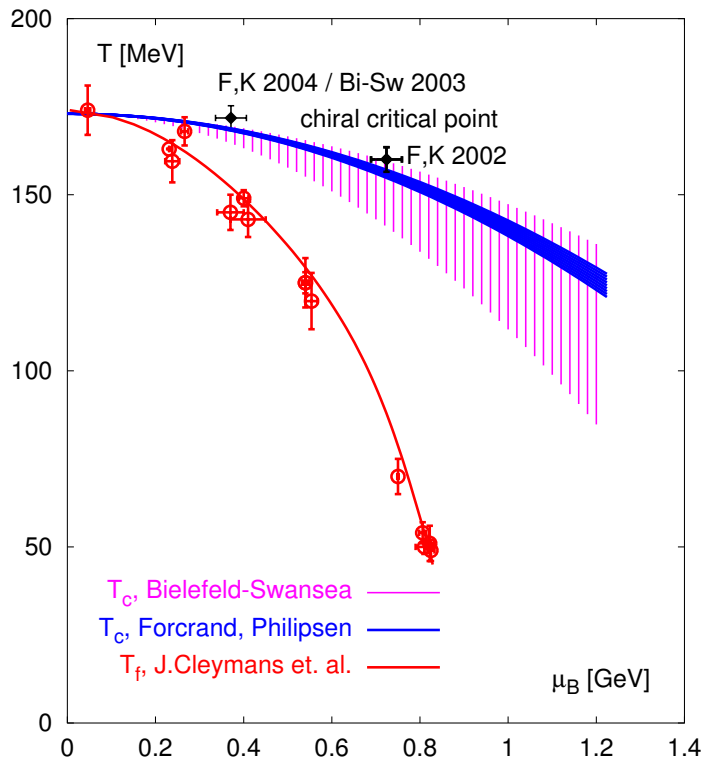
P. Braun-Munzinger, J. Stachel, C. Wetterich, hep-nucl/0311005



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$\begin{aligned}
 Z(V, T, \mu) &= \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E(V, T, \mu)} \\
 &= \int \mathcal{D}A \det M(\mu) e^{-S_E(V, T)}
 \end{aligned}$$



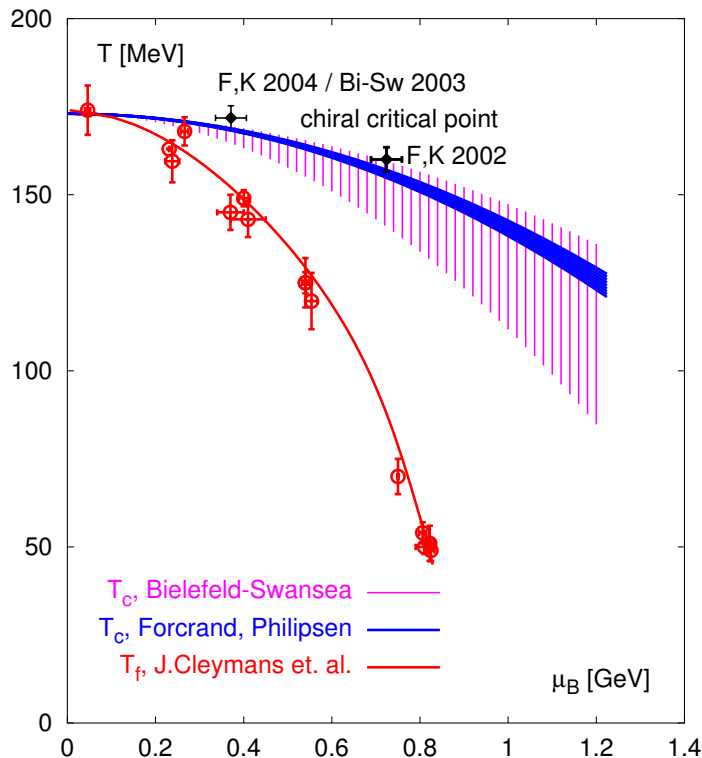
chiral critical point



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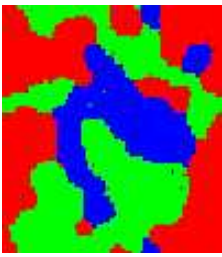
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chiral critical point

μ_c shifted from $\simeq 700 \text{ MeV}$ to $\simeq 400 \text{ MeV}$

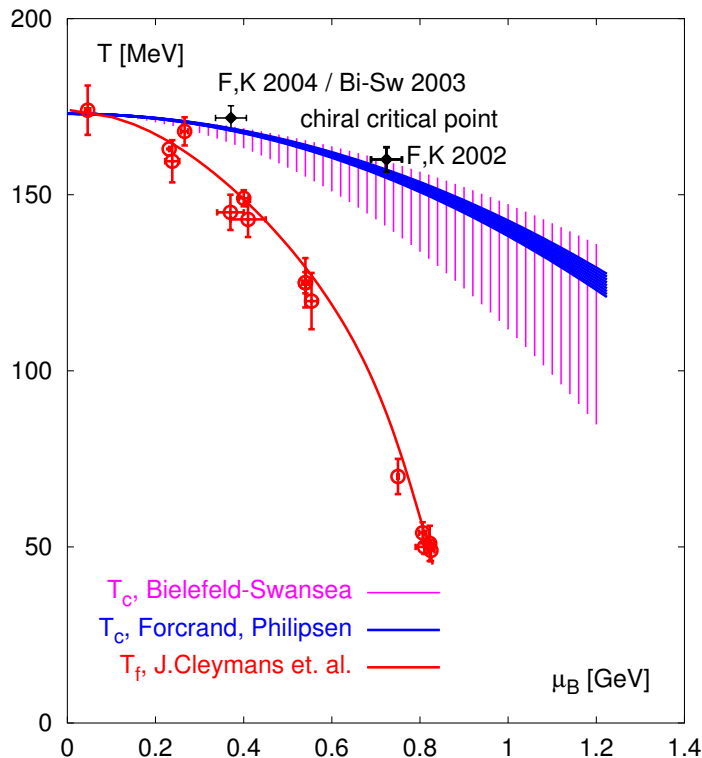
Fodor, Katz (exact determinants \Rightarrow small lattices)



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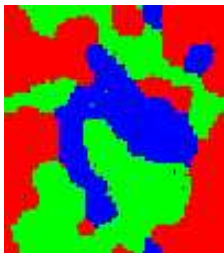


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$\mu_c(m_q)$: discrepancies between calculations with real and imaginary μ



Analyzing the (quasi-particle) structure of HG and QGP phases

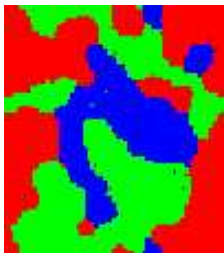
Response and correlation functions:

$T \leq T_c$: chiral symmetry restoration

- hadronic **resonance** gas;
MEM analysis of thermal masses and widths, π , ρ , ...
- (baryon) density fluctuations, strangeness fluctuations, ...

$T > T_c$: deconfinement

- free energies, potentials and **screening** masses, running coupling at short and large distances, ...
- **MEM analysis** of heavy and light quark bound states, quark and gluon propagators, dilepton and photon rates, ...



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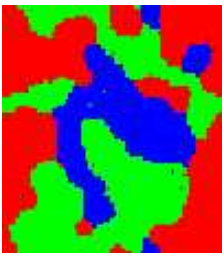
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requires light dynamical quarks
 \Rightarrow PETAFL0Ps era

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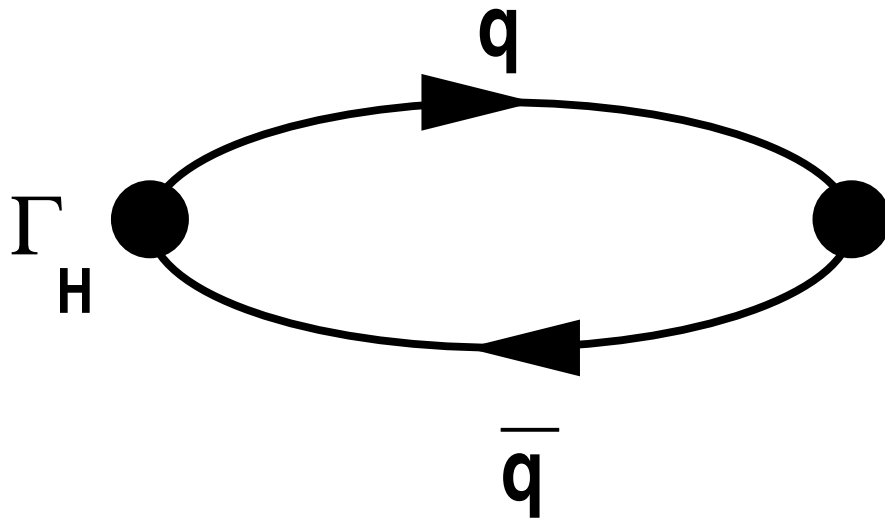
meaningful already in quenched QCD
 \Rightarrow TERAFL0Ps era



Thermal meson correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$ -pair

spectral representation of correlator \Rightarrow dilepton and photon rates



spectral representation of
Euclidean correlation functions

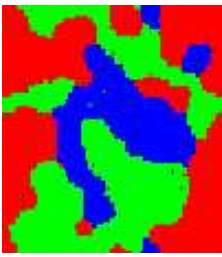
$$G_H^\beta(\tau, \vec{r}) = \int_0^\infty d\omega \int \frac{d^3\vec{p}}{(2\pi)^3} \sigma_H(\omega, \vec{p}, T) e^{i\vec{p}\vec{r}} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

spectral representation of
thermal photon rate: $\omega = |\vec{p}|$

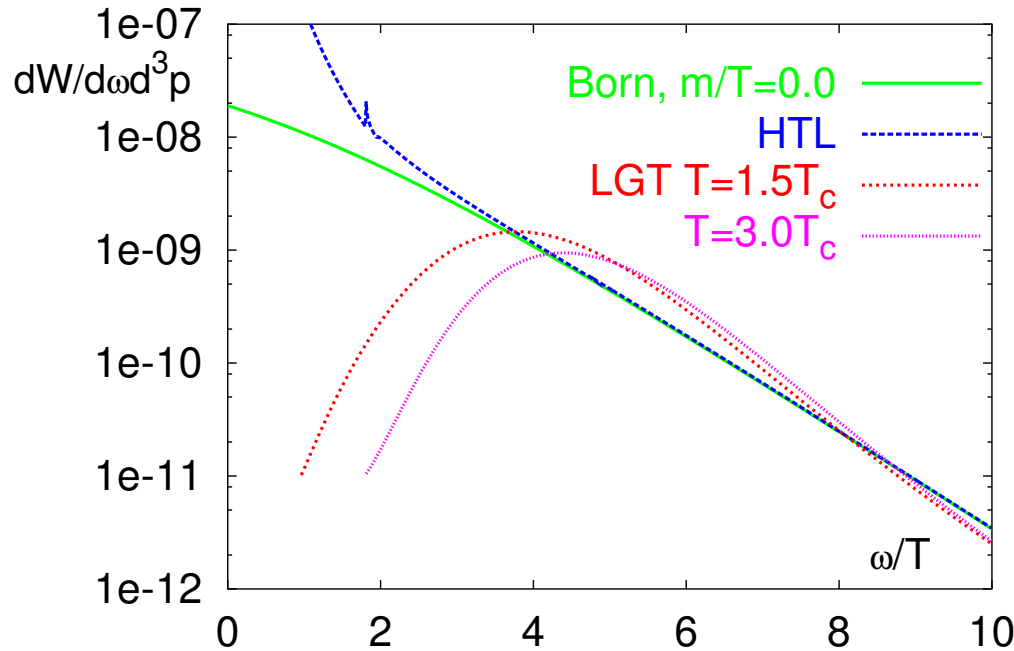
$$\omega \frac{d^3 R^\gamma}{d^3 p} = \frac{5\alpha}{6\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2 (e^{\omega/T} - 1)}$$

spectral representation of
thermal dilepton rate

$$\frac{d^4 W}{d\omega d^3 p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2 (e^{\omega/T} - 1)}$$



Dilepton rate: HTL and lattice calculations



thermal dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2 (e^{\omega/T} - 1)}$$

HTL and lattice disagree for
 $\omega/T \lesssim (3 - 4)$

- infra-red sensitivity of HTL-calculations \Leftrightarrow "massless gluon" cut in HTL-propagator
- infra-red sensitivity of lattice calculations \Leftrightarrow thermodynamic limit, $V \rightarrow \infty$
- $VT^3 = (N_\sigma/N_\tau)^3 < \infty \Rightarrow$ momentum cut-off: $p/T > 2\pi N_\tau/N_\sigma$



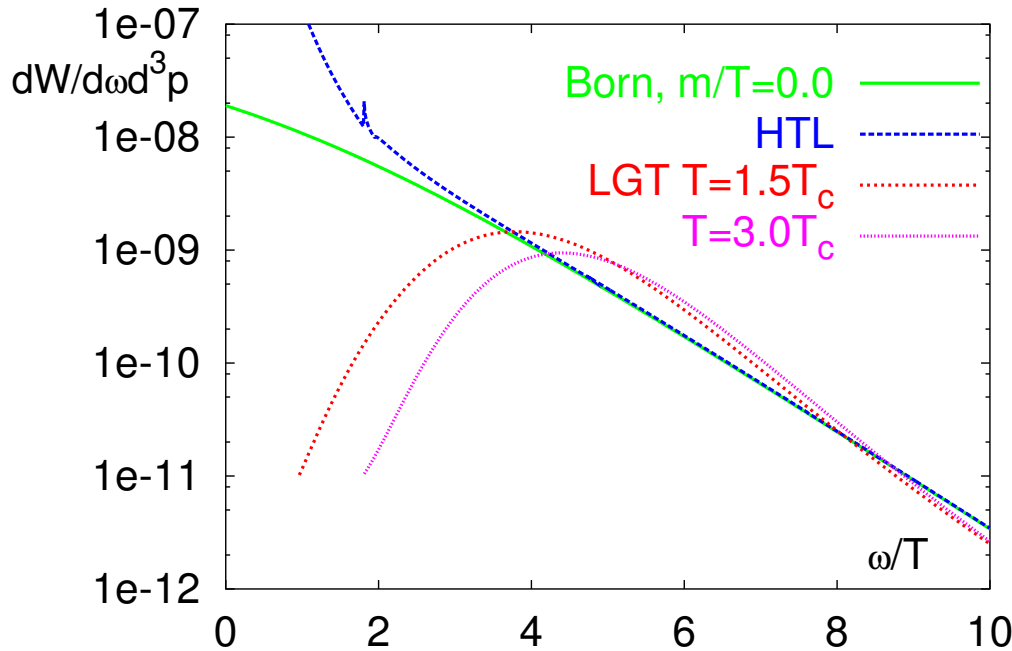
need large lattices to analyze infra-red regime



in future also thermal photon rates



Dilepton rate: HTL and lattice calculations



thermal dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2 (e^{\omega/T} - 1)}$$

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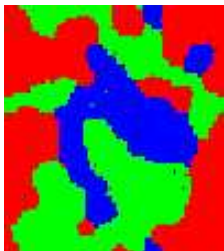


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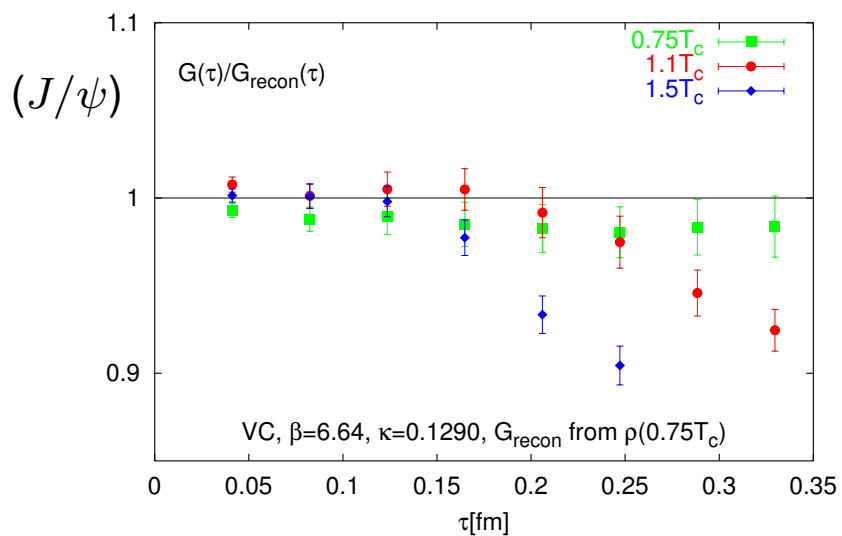
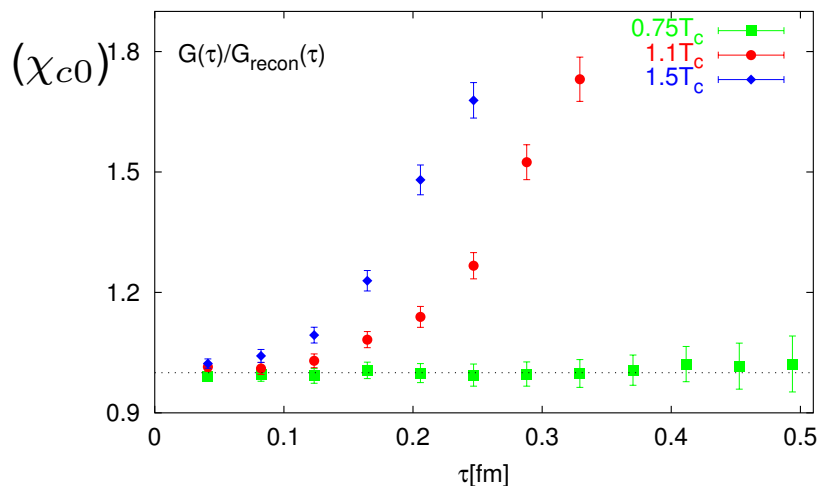
need $N_\tau \sim \mathcal{O}(30)$ AND
 $N_\sigma \sim 6 N_\tau$



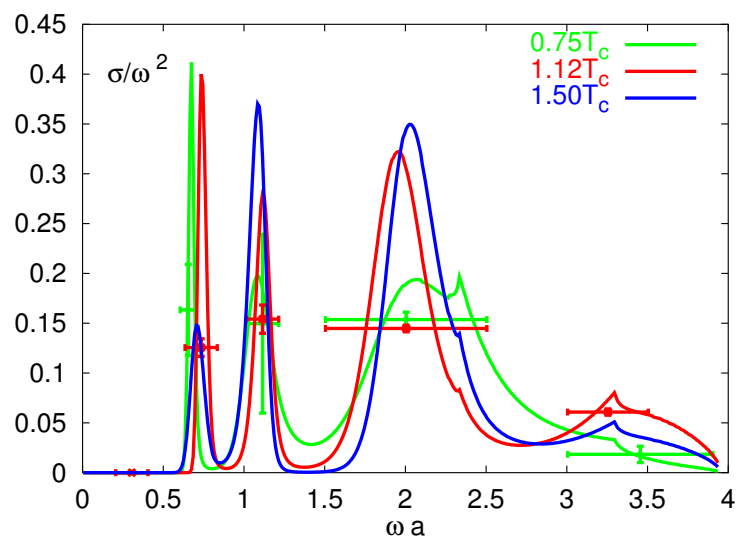
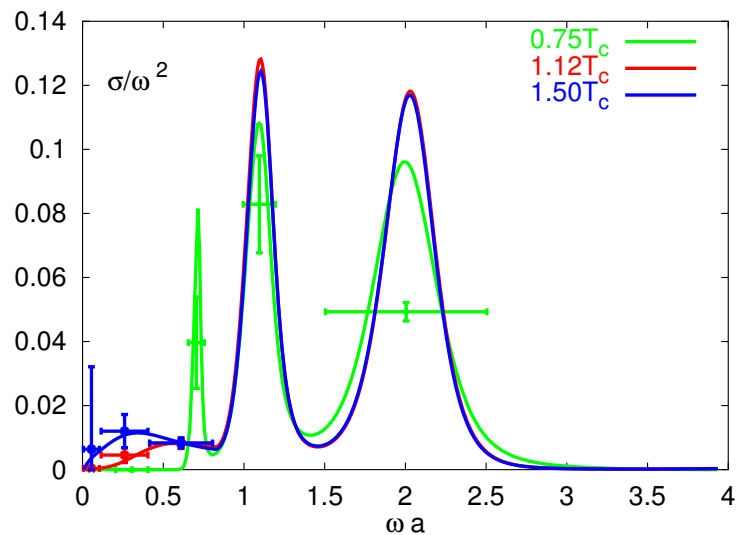
Heavy quark spectral functions and correlation functions

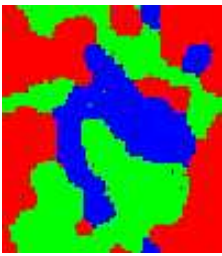
reconstructed correlation functions
above T_c from data below T_c

SC, $\beta=6.64$, $\kappa=0.1290$, G_{recon} from $\rho(0.75T_c)$



reconstructed spectral functions
using the Maximum Entropy Method

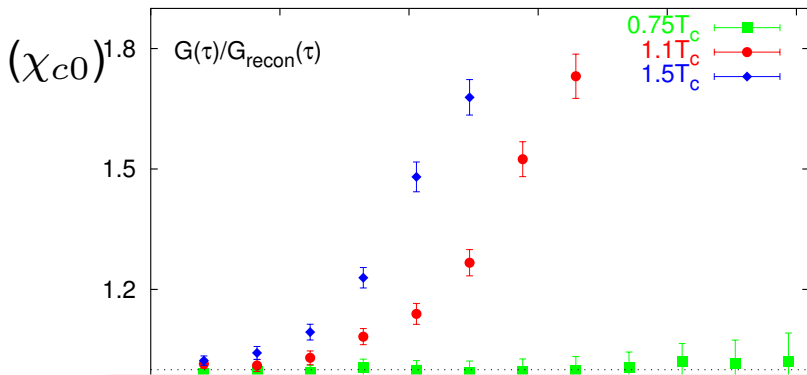




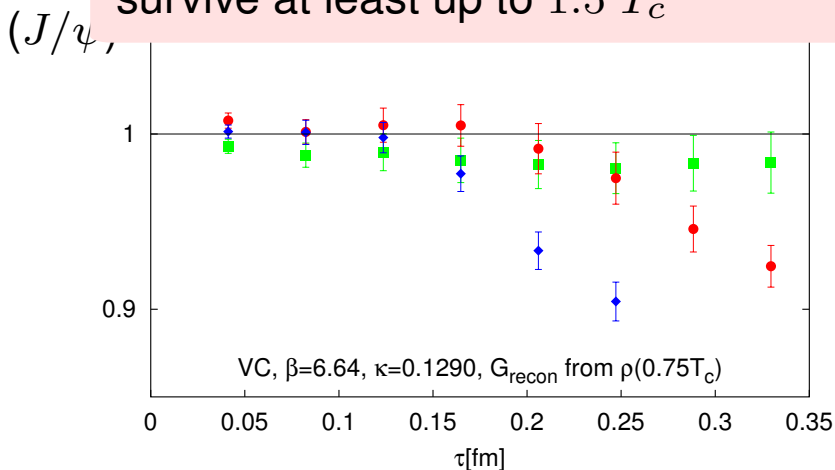
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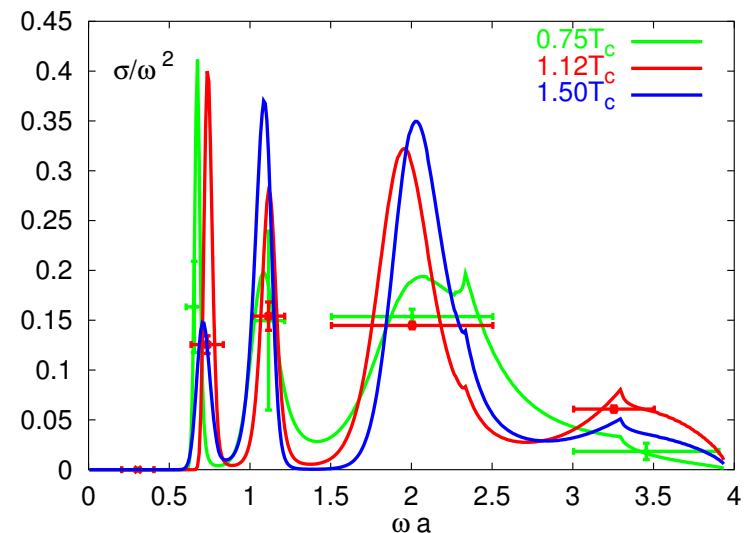
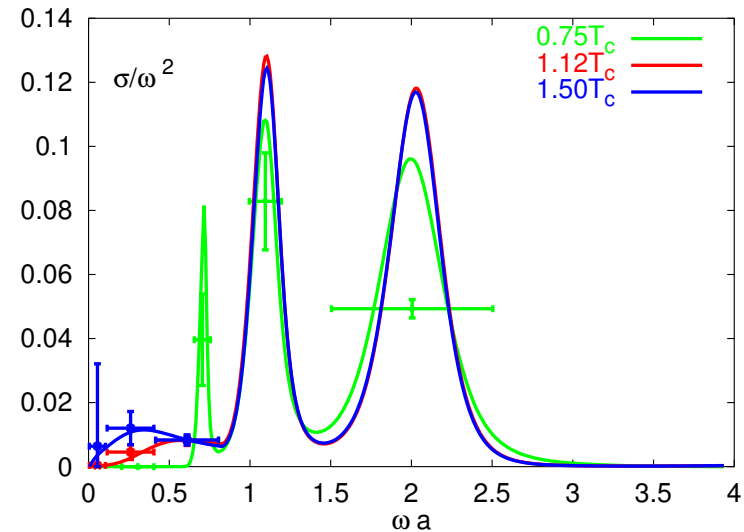
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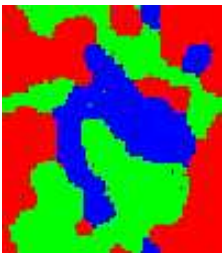


radial excitations (χ_c) disappear at T_c ;
charmonium S-states (J/ψ and η_c)
survive at least up to $1.5 T_c$



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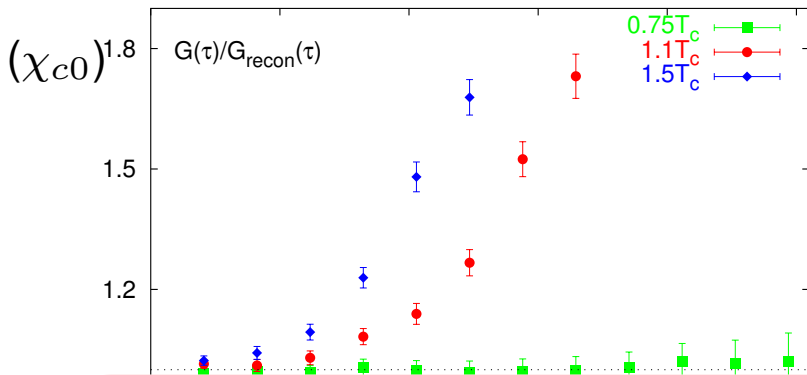




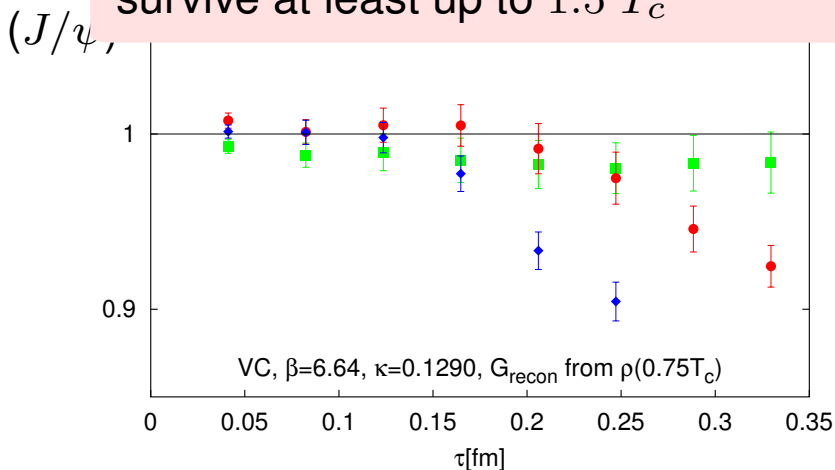
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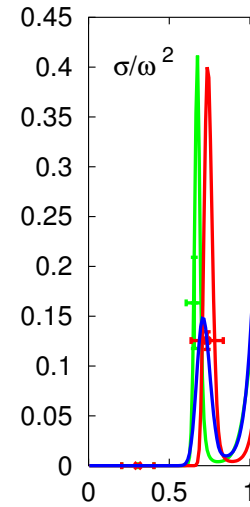
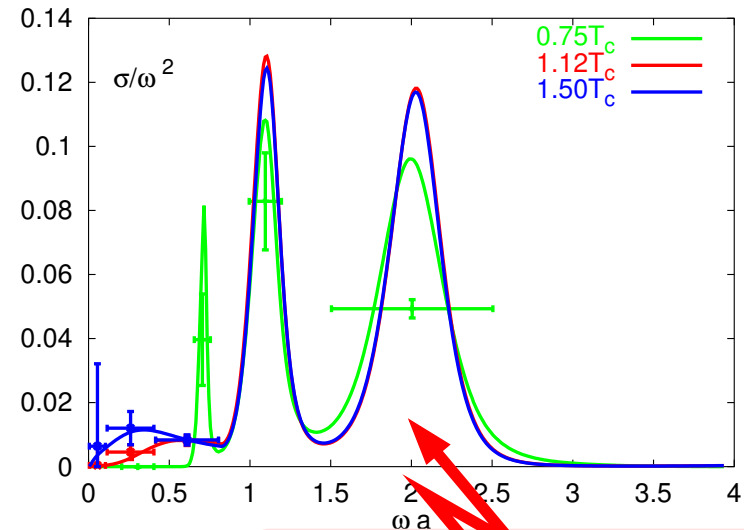
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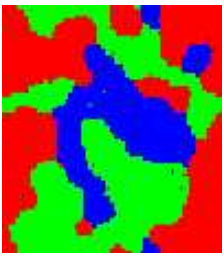
need to get better control over
ultra-violet cut-off effects
(Wilson-doublers)

use better fermion actions

- overlap fermions

- domain wall fermions

- (truncated) perfect actions...

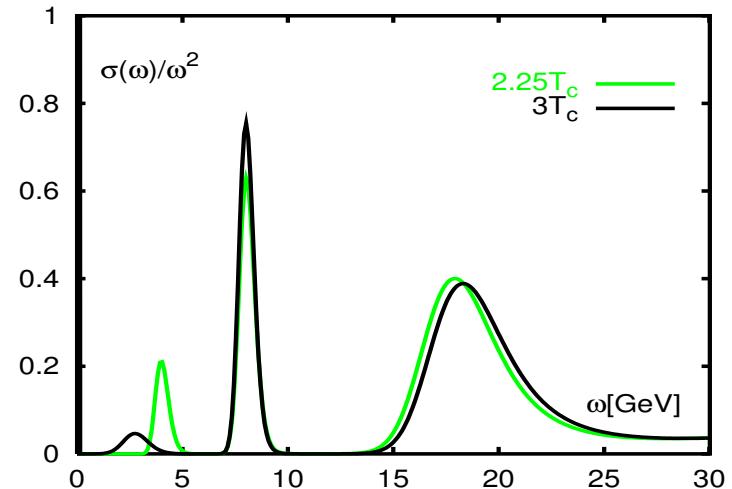
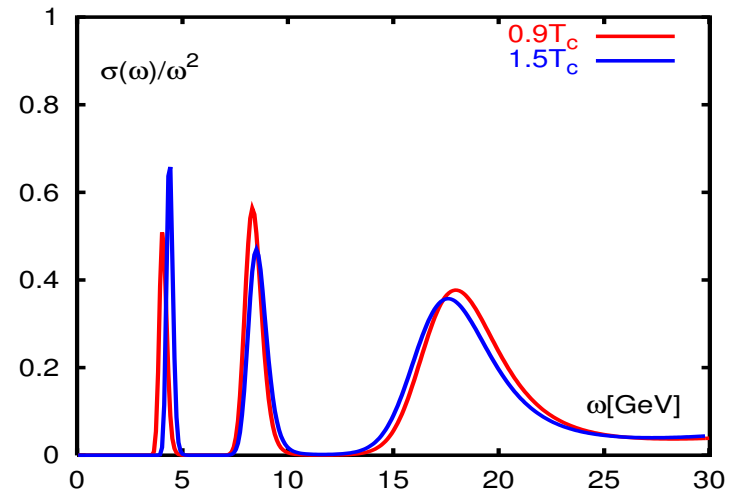
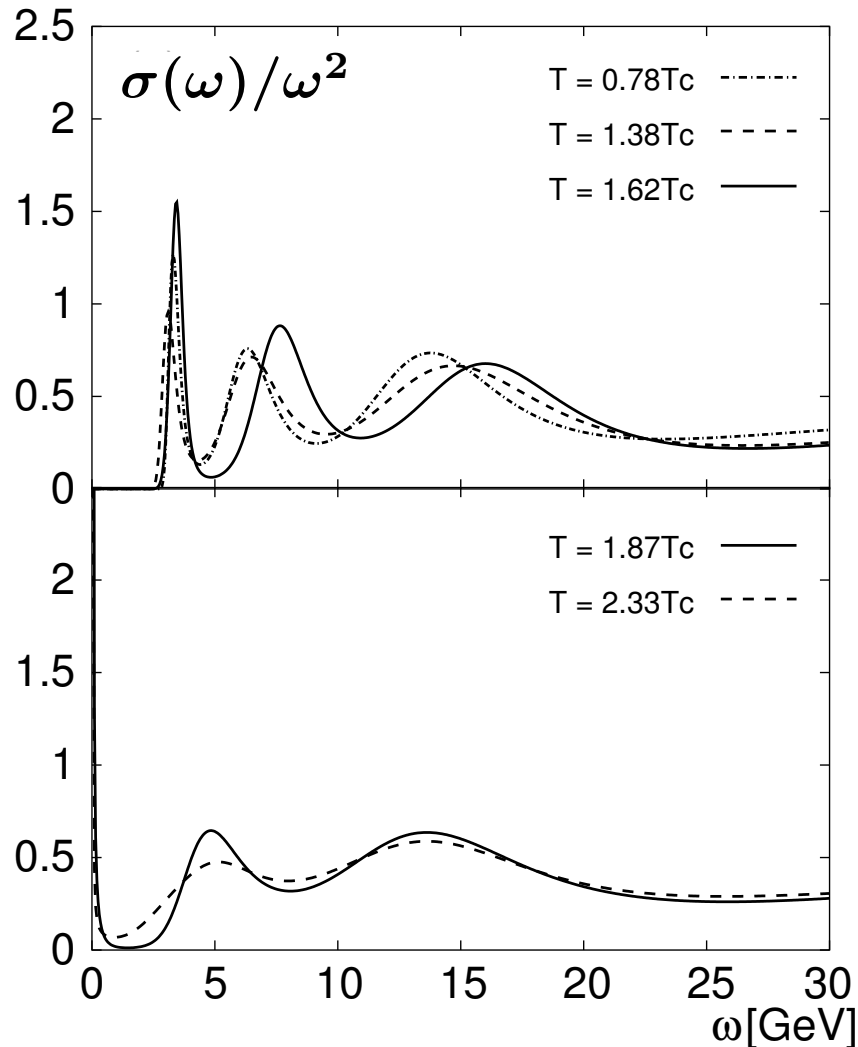


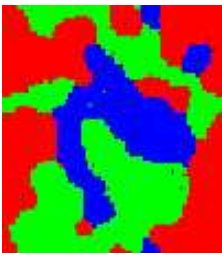
Heavy quark spectral functions comparison of different approaches

M. Asakawa, T. Hatsuda, hep-lat/0308034

S. Datta et al., hep-lat/0312037

J/ψ spectral function



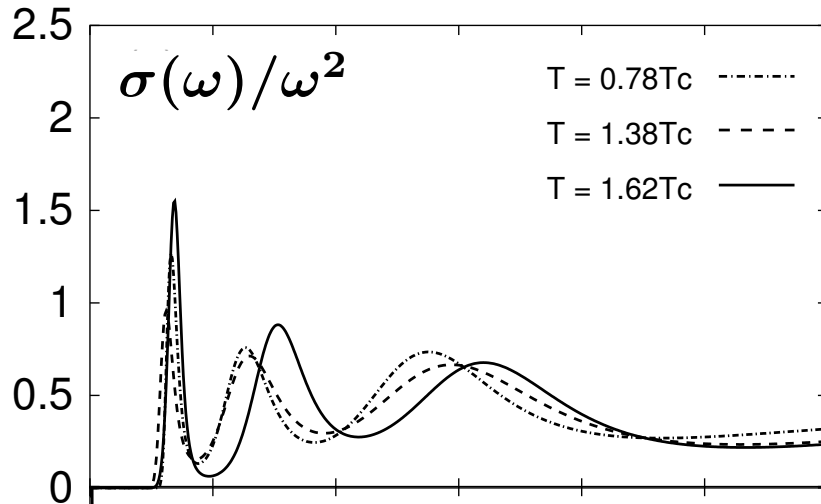


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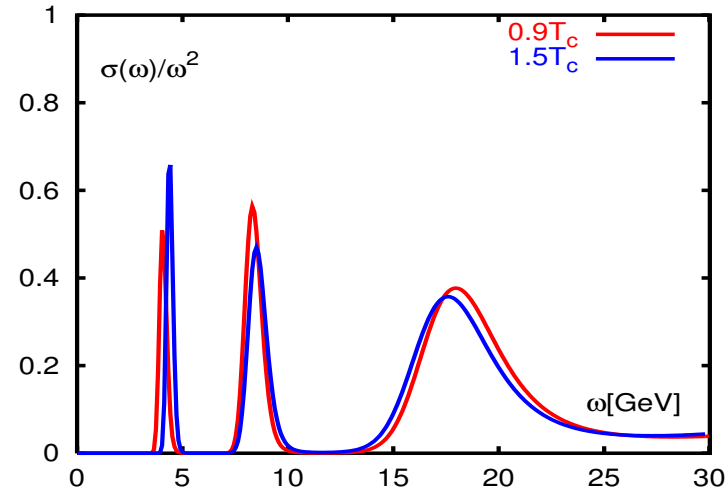
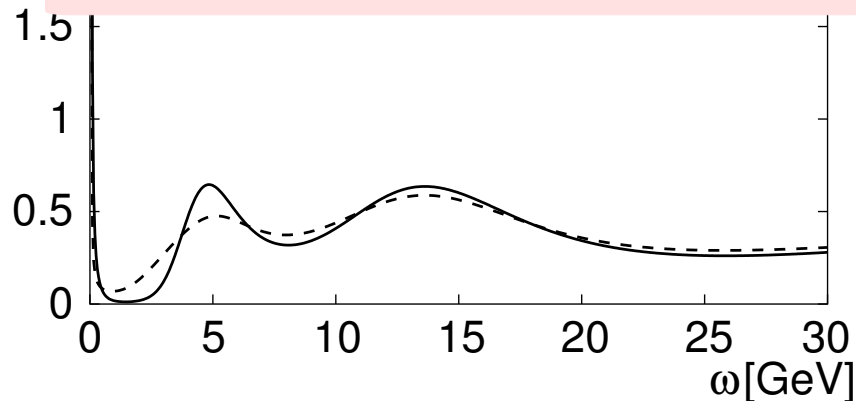
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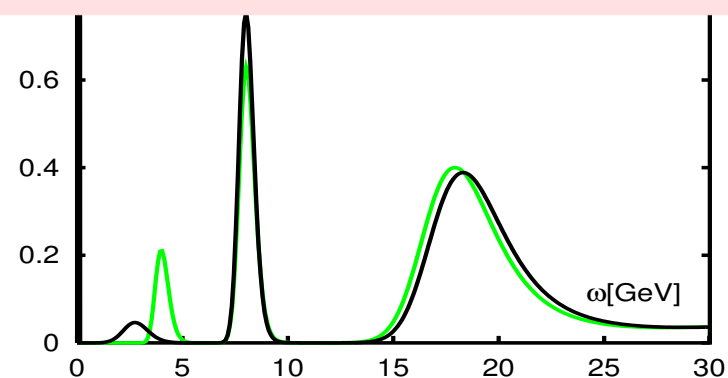
J/ψ spectral function

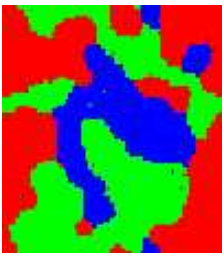


J/ψ dissociates for $1.6T_c \lesssim T \lesssim 1.9T_c$
rather abrupt disappearance of J/ψ



J/ψ gradually disappears for $T \gtrsim 1.5T_c$
 J/ψ strength reduced by 25% at $T = 2.25T_c$



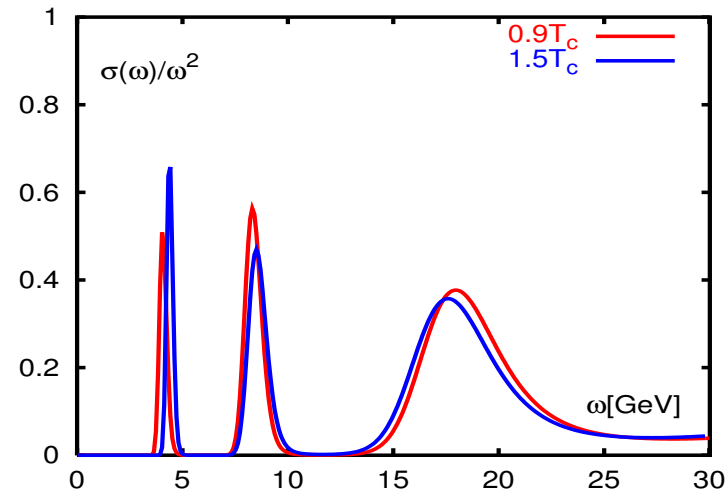
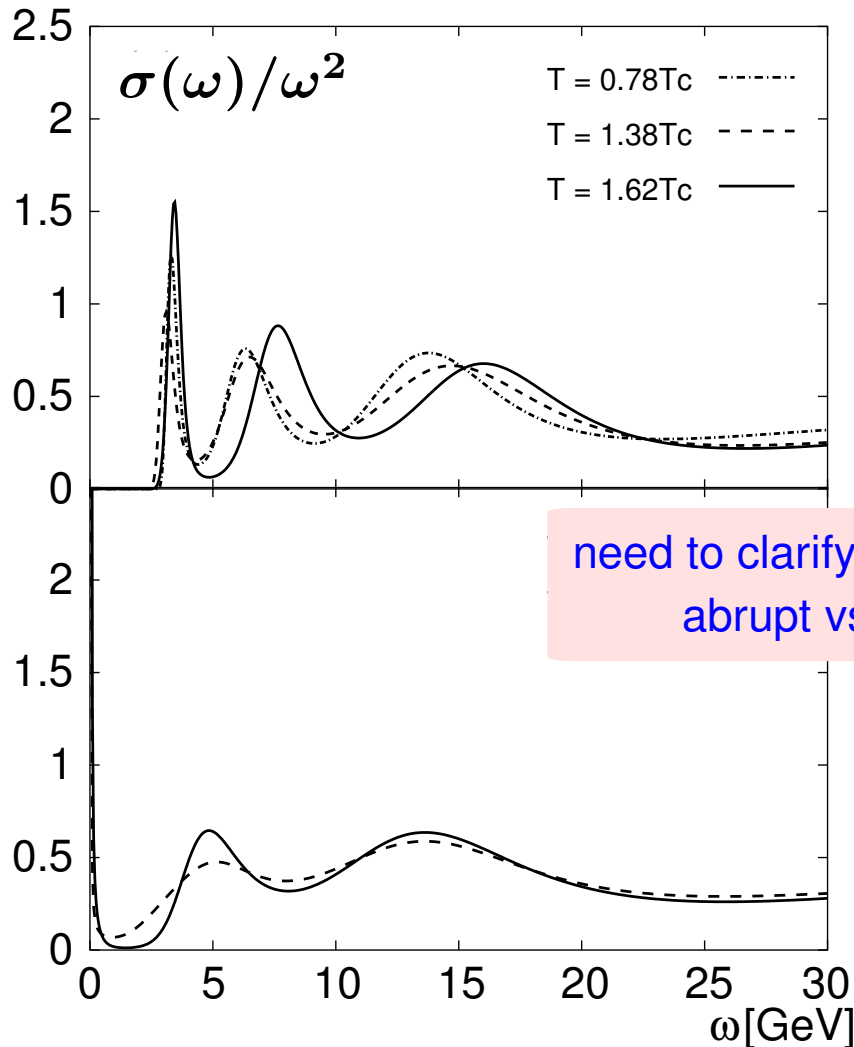


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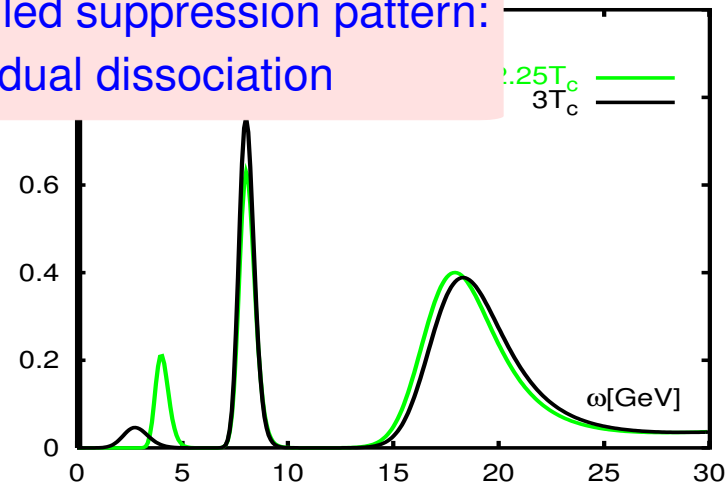
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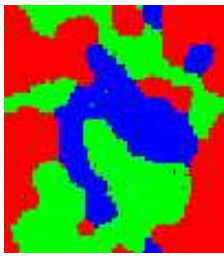
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J/ψ spectral function



need to clarify detailed suppression pattern:
abrupt vs. gradual dissociation



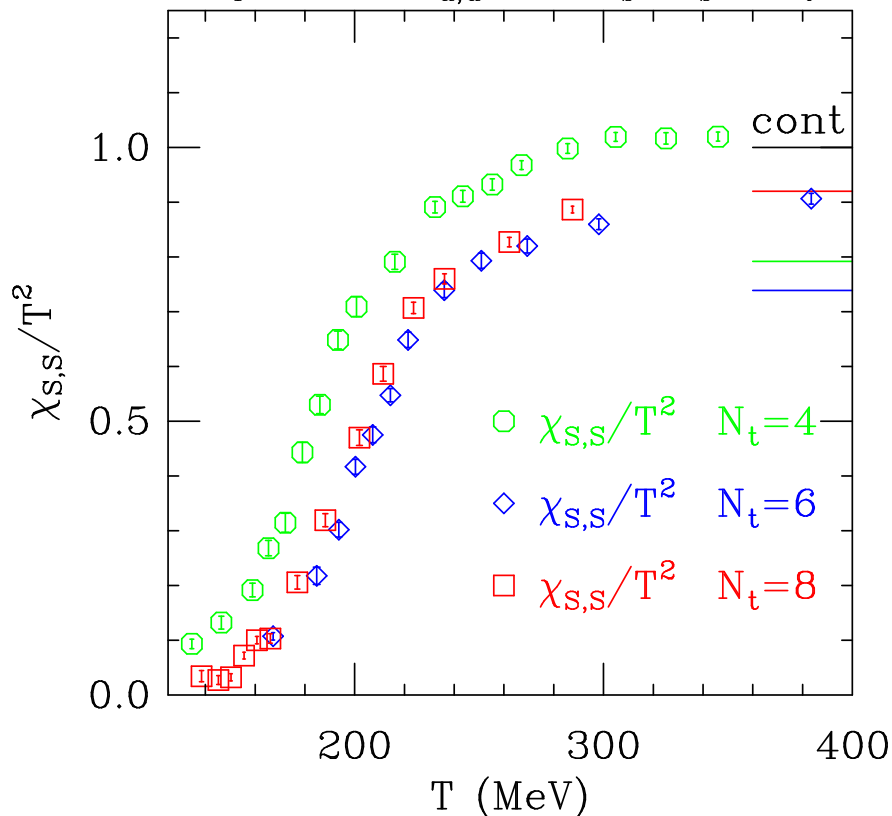


Fluctuations of the baryon number density ($\mu \geq 0$)

baryon number density fluctuations:
(MILC coll., hep-lat/0405029)

$$\mu = 0$$

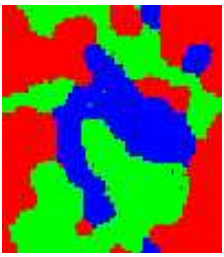
$$N_f=2+1, m_{u,d}=0.2m_s, N_s=2N_t$$



$$\begin{aligned} \frac{\chi_q}{T^3} &= \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}} \\ &= \frac{9 T}{V} (\langle N_B^2 \rangle - \langle N_B \rangle^2) \end{aligned}$$

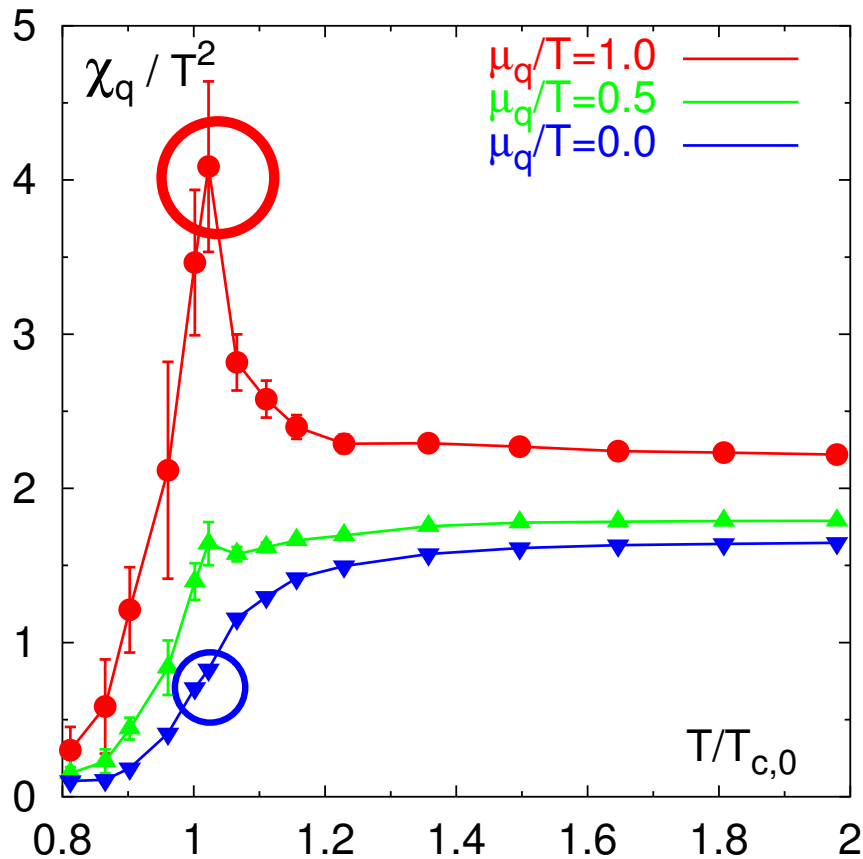
susceptibilities = integrated correlation functions
= integrated spectral functions

to be studied in event-by-event fluctuations



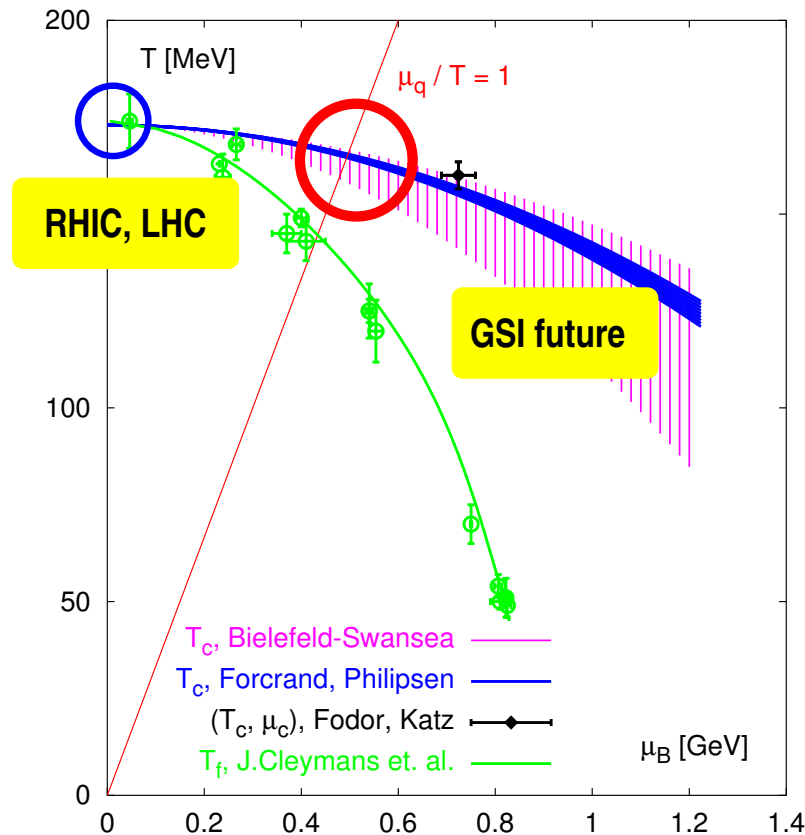
Fluctuations of the baryon number density ($\mu \geq 0$)

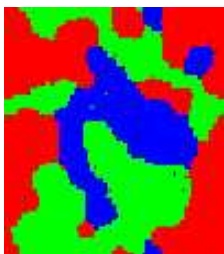
baryon number density fluctuations:
 (Bielefeld-Swansea, PRD68 (2003) 014507)
 $\mu \geq 0, n_f = 2$



$$\frac{\chi_q}{T^3} = \left(\frac{d^2 p}{d(\mu/T)^2 T^4} \right)_{T \text{ fixed}}$$

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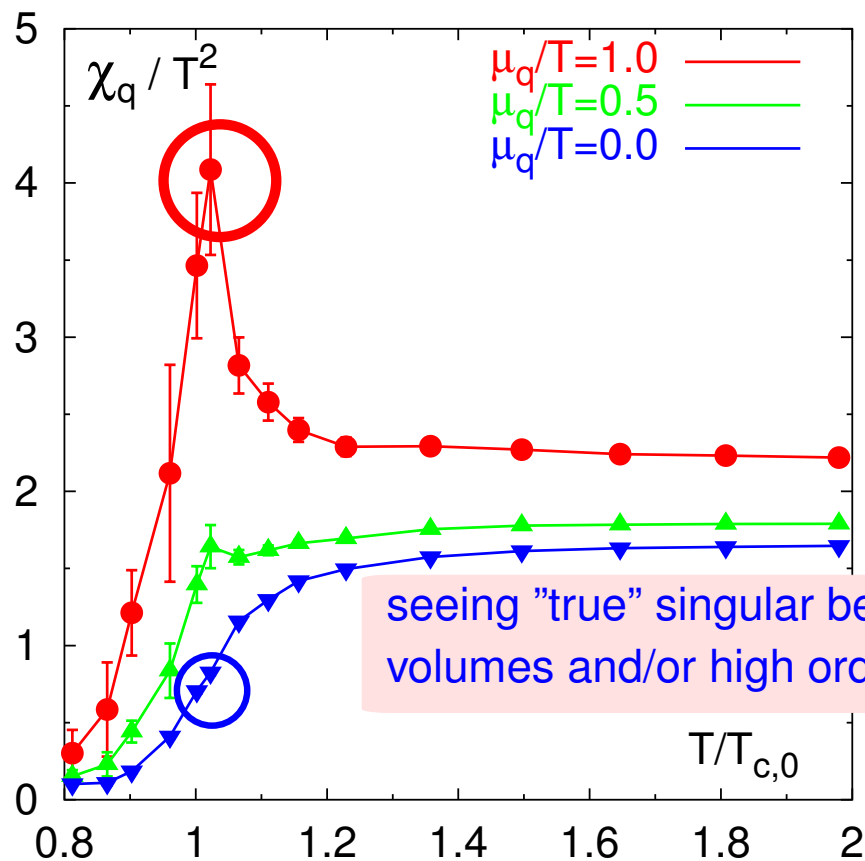




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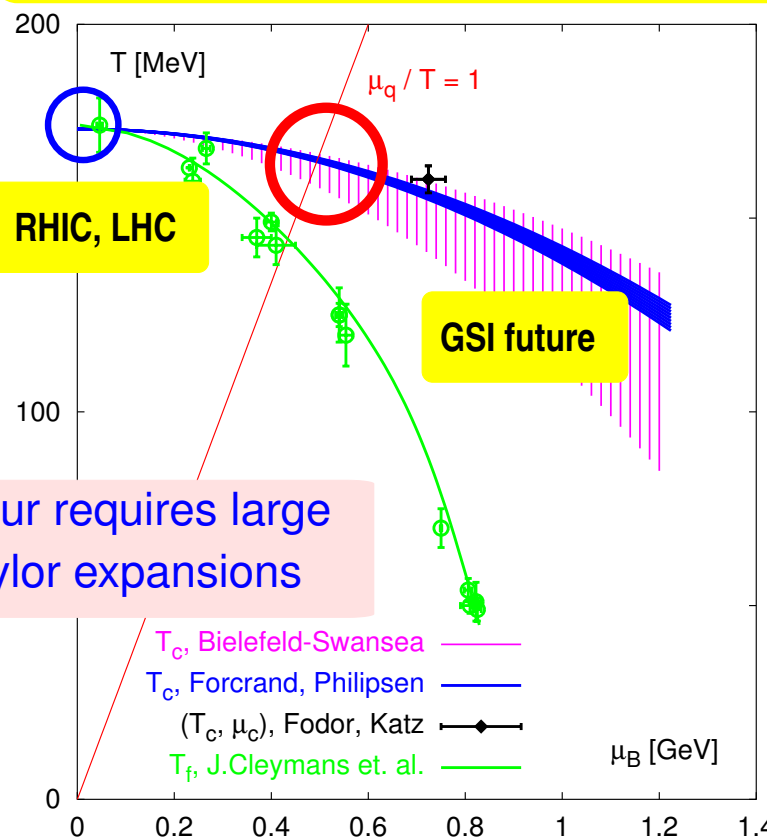
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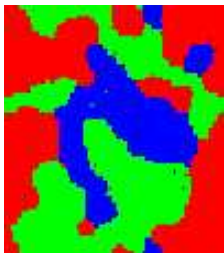
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Outlook: Next generation lattice calculations

- Thermodynamics of pure gauge theory has been "solved" on (1-10)GFlops computers (1996)
- Thermodynamics of QCD with "still too heavy" quarks has been studied on (10-100) GFlops computers
- Analysis of "continuum and thermodynamic limit" of bulk thermodynamics with light quarks and spectral functions in quenched QCD requires computers with ~ 10 TFlops peak speed.

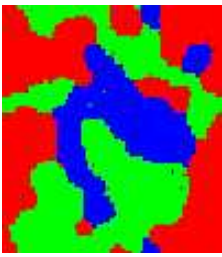
Germany: LatFor proposal 2003

<http://www.zeuthen.desy.de/latfor/paper.pdf>

US: White Paper 2004

<http://www-ctp.mit.edu/~negele/WhitePaper.pdf>

- Studies of spectral functions of light quark bound states below T_c require simulations with light, dynamical quarks on computers with $\gtrsim 100$ TFlops peak speed.



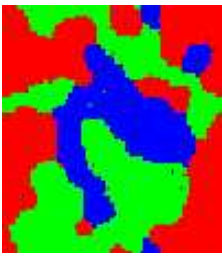
Outlook: projects coming soon...

Thermodynamics on a 10 TFlops computer (5 TFlops sustained)

- T_c , EoS ($\mu = 0$ and $\mu > 0$) with light dynamical quarks:
(2+1)-flavor QCD, close to physical m_π/m_K ratio;
exploring the continuum limit: $a \simeq (0.1 - 0.2)$ fm
analyzing the thermodynamic limit: $V \simeq 500 \text{ fm}^3$

 \Rightarrow lattice sizes up to: $32^3 \times 8$; CPU-time: ~ 5 TFlops-years ($\mu = 0$)
 ~ 5 TFlops-years ($\mu > 0$)
- In-medium hadron properties, charmonium, dilepton rates:
quenched QCD on fine lattices ($a \simeq 0.02$ fm);
analyzing light quark mesons with improved fermion formulations;
exploring infra-red sensitivity of dilepton rates;
analyzing charmonium spectra;

 \Rightarrow lattice sizes up to: $128^3 \times 32$; CPU-time: ~ 3 TFlops-years



Outlook: projects on future machines...

Thermodynamics on $\gtrsim 100$ TFlops computers

(exploratory studies already on up-coming TFlops computers)

- **In-medium properties of light quark bound states:**
QCD with light, dynamical quarks on fine lattices become possible;
mass shifts and modification of widths below T_c
- **finite density QCD at low temperature:**
temperatures around $T \sim 0.5 T_c$ should be accessible
- **transport properties:**
calculation of "gluonic correlator" (energy momentum tensor) become possible;
spectral functions in the $\omega \rightarrow 0$ limit will be analyzed